Coordinated Relay Beamforming for Amplify-and-Forward Two-Hop Interference Networks

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***Abstract*—Relaying is a promising technique to extend coverage and improve throughput in wireless networks, but its performance is degraded in the presence of co-channel interference. In this paper, we consider coordinated relay beamforming to suppress interference and improve the date rates of two-hop interference networks. We first propose optimal coordinated relay beamforming algorithms to characterize the achievable rate region and maximize the sum-rate. By imposing a constraint on the desired signals, a low-complexity iterative algorithm is then proposed to maximize the sum-rate. Through performance comparison, we show that the proposed relaying strategy provides a promising tradeoff between complexity and performance. To further reduce design complexity, we propose a new interference management scheme, *interference neutralization*, to cancel the interferences over the air at the second hop. We show that this scheme yields a closed-form solution for the beamforming design and provides good performance especially at high signal-to-noise ratio (SNR).**

I. Introduction

Relaying is considered as an efficient way to enlarge the coverage and increase the system capacity in wireless networks. Recently, various relaying protocols have been included in major future cellular communication systems, such as 3GPP LTE-Advanced and WiMAX [1]. Among the different relaying protocols, the amplify-and-forward (AF) scheme is popular for its low complexity and implementation cost [2]. In this paper, we consider two-hop relaying networks with multiple signal-antenna source-destination pairs communicating through multiple multi-antenna AF relays. This setting can arise in many systems, e.g., in the LTE-Advanced cellular system, where multi-antenna relays will be employed to improve the throughput and extend the coverage of the network. However, the added relays in these systems make the interference problem more complicated. In this paper, we will focus on interference mitigation via coordinated relay beamforming, since interference will be a limiting factor in future wireless networks.

Recently, there have been significant progresses on coordinated beamforming for single-hop MISO interference channels to mitigate inter-cell interferences [3], [4]. However, the understanding of relay beamforming for two-hop interference networks is very limited. The existing works are mainly focused on the total relay power minimization problem subject to QoS constraints at each destination node [5], or the total leakage (i.e., total interference power plus total forwarded noise power from the first hop) minimization algorithm subject to signal preservation constraints [6] or a sum relay power constraint [7]. In this paper, we will propose efficient techniques to maximize the sum-rate and characterize the achievable rate region with coordinated relay beamforming. Our results on the optimal relay beamforming will provide performance benchmarks for designing practical low-complexity algorithms.

Motivated by the high complexity of the optimal beamforming, our second objective is to find an efficient approach to maximize the sum-rate. We shall propose a novel method, which aligns the desired signals at different destinations at the same level, instead of canceling the interference. The proposed iterative algorithm based on this strategy will be shown to provide better performance compared with the existing suboptimal relay beamforming schemes. This provides a new approach to manage the interference for two-hop networks in a more tractable way.

Traditionally, zero-forcing (ZF) beamforming is well known to reduce the design complexity. Due to its simplicty, ZF beamforming is a promising transmission technique in the single-hop MISO interference channel [8]. In [9], Zhang and Letaief proposed ZF-based approaches for two-hop decode-and-forward (DF) relay networks, which provide a promising tradeoff between the complexity and the achievable performance. Unfortunately, such complexity advantage of the ZF approach cannot be carried over to two-hop AF relay networks. Actually, for AF relay networks, we need to deal with not only interferences but also the forwarded noises from the first hop. Moreover, it is impossible to cancel the forwarded noise via the ZF approach. Liu and Petropulu [10] considered the sum-rate maximization problem using ZF beamforming in the relay interference network with a single multi-antenna AF relay, but the proposed algorithm can not be extended to the multiple relay case.

Interference neutralization, a close idea to ZF beamforming, was recently shown to be a powerful scheme to optimize the degrees of freedom (DoF) for the two-hop interference relay networks [11]. This scheme allows interference to be canceled over the air at the destinations. However, its performance has not yet been investigated in the finite SNR region via

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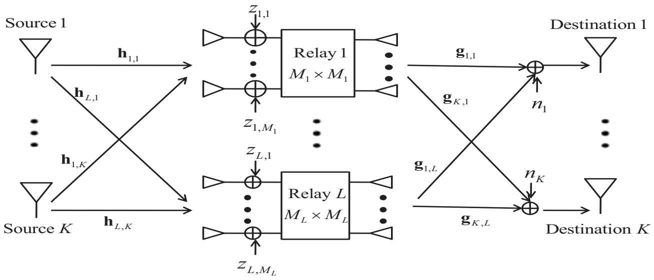

Fig. 1. A two-hop interference channel with K single-antenna source-destination pairs and L AF relays.

the coordinated relay beamforming design. In this paper, from the achievable rate region comparison, we will show that this scheme provides good performance with a relatively low complexity. Furthermore, we will derive a closed-form solution to the interference neutralization beamforming, which is shown to provide good performance at high SNR values.

Notations: Boldface lower case and upper case letters represent vectors and matrices, respectively. The superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, $(\cdot)^{-1}$ denote the transpose, the complex conjugate, the Hermitian, the matrix inverse operators. $\mathbf{1}_n$, $\mathbf{0}_n$, $\mathbf{I}$, and $\mathbf{0}$ denote the n-dimensional column vectors with ones, zeros, the identity matrix and the all-zero matrix, respectively. $\mathrm{Tr}\{\cdot\}$, $\Re\{\cdot\}$, $|\cdot|$ and $||\cdot||$ denote the trace operator, the real part, the absolute value and the standard Euclidean norm. $\mathbb{E}\{\cdot\}$ denotes the statistical expectation. Finally, $\mathbf{S} \succeq 0$ means that $\mathbf{S}$ is a positive semi-definite matrix.

II. System Model and Problem Formulations

We consider a system consisting of K single-antenna source-destination pairs communicating through L AF relays as shown in Fig. 1. The lth relay is equipped with M_l antennas that are used for receiving signals from the sources and transmitting signals to the destination terminals. The relays operate in the half-duplex mode, i.e., the signals are transmitted from the sources to the destinations through two phases. In the first phase, the sources transmit signals to the relays, and in the second phase, the relays transmit the processed signals to the destination terminals. The $M_l\times 1$ received signal vector $\mathbf{r}_l$ at the lth relay can be written as

$$\mathbf{r}_l = \mathbf{H}_l\mathbf{P}^{1/2}\mathbf{s} + \mathbf{z}_l, \tag{1}$$

where $\mathbf{z}_l \triangleq [z_{l,1}, ..., z_{l,M_l}]^T$ is the $M_l\times 1$ vector of white Gaussian noise with distribution $\mathcal{CN}(\mathbf{0}, \sigma_r^2\mathbf{I})$, $\mathbf{s} \triangleq [s_1, ..., s_K]^T \in \mathbb{C}^{K\times 1}$ with s_k as the transmitted symbol from the kth source with $\mathbb{E}\{|s_k|^2\} = 1$, $\mathbf{P} \triangleq \mathrm{diag}\{p_1, ..., p_K\} \in \mathbb{R}^{K\times K}$ with p_k as the transmit power of the kth source (we assume that the sources use full power transmission as we restrict our interest to relay beamforming), $\mathbf{H}_l \triangleq [\mathbf{h}_{l,1}, ..., \mathbf{h}_{l,K}] \in \mathbb{C}^{M_l\times K}$, where $\mathbf{h}_{l,k}$ is the $M_l \times 1$ vector containing the channel coefficients from the kth source to the lth relay. In the second phase, the lth relay retransmits the received signal in (1) after multiplying the beamforming matrix $\mathbf{W}_l$. As a result, the signal transmitted from the lth relay can be written as

$$\mathbf{t}_l = \mathbf{W}_l\mathbf{H}_l\mathbf{P}^{1/2}\mathbf{s} + \mathbf{W}_l\mathbf{z}_l. \tag{2}$$

Using (1) and (2), we can write the signal received at the kth destination as

$$y_k = \underbrace{\sum_{l=1}^{L}\sqrt{p_k}\mathbf{g}_{k,l}^T\mathbf{W}_l\mathbf{h}_{l,k}s_k}_{y_{S,k}} + \underbrace{\sum_{l=1}^{L}\mathbf{g}_{k,l}^T\mathbf{W}_l\tilde{\mathbf{H}}_{l,k}\tilde{\mathbf{P}}_k^{1/2}\tilde{\mathbf{s}}_k}_{y_{I,k}} + \underbrace{\sum_{l=1}^{L}\mathbf{g}_{k,l}^T\mathbf{W}_l\mathbf{z}_l}_{y_{N,k}} + n_k, \tag{3}$$

where n_k is the additive noise at the kth destination with distribution $\mathcal{CN}(0, \sigma_d^2)$, $\mathbf{g}_{k,l}$ is the $M_l \times 1$ vector consisting of the channel coefficients between the lth relay and the kth destination, $\tilde{\mathbf{s}}_k \triangleq [s_1, ..., s_{k-1}, s_{k+1}, ..., s_K]^T$ is the $(K-1)\times 1$ vector consisting of the signals transmitted by the sources that are not targeting the kth destination, $\tilde{\mathbf{P}}_k \triangleq \mathrm{diag}\{p_1,...,p_{k-1},p_{k+1},...,p_K\}$ consists of the transmit powers of the corresponding sources, and $\tilde{\mathbf{H}}_{l,k} \triangleq [\mathbf{h}_{l,1}, ..., \mathbf{h}_{l,k-1}, \mathbf{h}_{l,k+1}, ..., \mathbf{h}_{l,K}]$ consists of the corresponding source-relay channels. Therefore, $y_{S,k}$, $y_{I,k}$, $y_{N,k}$ are the desired signal, interference signals and forwarded noise components at the kth receiver, respectively. Throughout the paper, we will assume that each relay has the perfect global channel state information (CSI) from all the sources to relays and all relays to destinations. Furthermore, we assume that transmission symbols, relay noises and destination noises are mutually statistically independent.

Using the identity $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{B})$, where $\mathrm{vec}\{\cdot\}$ is the vectorization operator that stacks the columns of a matrix on the top of each other and $\otimes$ denotes the Kronecker product operator, the transmit power at the lth relay is given by $P_{R_l} = \mathbb{E}\{||\mathbf{t}_l||^2\} = \mathbf{w}_l^H\mathbf{D}_l\mathbf{w}_l$, where $\mathbf{w}_l = \mathrm{vec}(\mathbf{W}_l)$ is the $M_l^2 \times 1$ beamforming vector and $\mathbf{D}_l \triangleq (\mathbf{H}_l\mathbf{P}^{1/2})^*(\mathbf{H}_l\mathbf{P}^{1/2})^T \otimes \mathbf{I}_{M_l} + \sigma_r^2\mathbf{I}_{M_l^2}$ is the $M_l^2 \times M_l^2$ matrix. As a result, the sum-power at all relays can be written as $P_R = \sum_{l=1}^{L} P_{R_l} = \mathbf{w}^H\mathbf{D}\mathbf{w}$, where $\mathbf{w} \triangleq [\mathbf{w}_1^T,...,\mathbf{w}_L^T]^T$ is the $\sum_{l=1}^L M_l^2 \times 1$ stacked relay beamforming vector and $\mathbf{D} \triangleq \mathrm{diag}\{\mathbf{D}_1,...,\mathbf{D}_L\}$ is an $\sum_{l=1}^L M_l^2 \times \sum_{l=1}^L M_l^2$ diagonal matrix. Throughout this paper, we will assume that the cooperative relay beamforming matrices are designed subject to a sum-power constraint [7] for all relays

$$\mathbf{w}^H\mathbf{D}\mathbf{w} \le P, \tag{4}$$

where P is the maximum total transmit power at all relays.

From (3), we can derive expressions for the desired signal power $P_{S,k}$, interference signal power $P_{I,k}$, and forwarded noise power $P_{N,k}$ at the kth destination as follows. $P_{S,k} = \mathbb{E}\{|y_{S,k}|^2\} = p_k|\mathbf{b}_k^H\mathbf{w}|^2$, where $\mathbf{b}_k \triangleq [(\mathbf{h}_{1,k}^T \otimes \mathbf{g}_{k,1}^T), ..., (\mathbf{h}_{L,k}^T \otimes \mathbf{g}_{k,L}^T)]^H$ is the $\sum_{l=1}^L M_l^2 \times 1$ vector. $P_{I,k} =$

$\mathbb{E}\{|y_{I,k}|^2\} = ||\mathbf{A}_k\mathbf{w}||^2 = \mathbf{w}^H\mathbf{Q}_k\mathbf{w}$, where $\mathbf{Q}_k \triangleq \mathbf{A}_k^H\mathbf{A}_k$ is the $\sum_{l=1}^L M_l^2 \times \sum_{l=1}^L M_l^2$ matrix with the $(K-1)\times\sum_{l=1}^L M_l^2$ matrix $\mathbf{A}_k \triangleq [(\tilde{\mathbf{H}}_{1,k}\tilde{\mathbf{P}}_k^{1/2})^T \otimes \mathbf{g}_{k,1}^T, ..., (\tilde{\mathbf{H}}_{L,k}\tilde{\mathbf{P}}_k^{1/2})^T \otimes \mathbf{g}_{k,L}^T]$. $P_{N,k} = \mathbb{E}\{|y_{N,k}|^2\} = ||\mathbf{B}_k\mathbf{w}||^2 = \mathbf{w}^H\mathbf{G}_k\mathbf{w}$, where $\mathbf{B}_k = \text{diag}\{\mathbf{B}_{1,k}, ..., \mathbf{B}_{L,k}\}$ is the $\sum_{l=1}^L M_l \times \sum_{l=1}^L M_l^2$ diagonal matrix with the $M_l \times M_l^2$ matrix $\mathbf{B}_{l,k} \triangleq (\sigma_r\mathbf{I}_{M_l}) \otimes \mathbf{g}_{k,l}^T$ and $\mathbf{G}_k \triangleq \mathbf{B}_k^H\mathbf{B}_k$ is the $\sum_{l=1}^L M_l^2 \times \sum_{l=1}^L M_l^2$ matrix.

In this paper, we consider the single-user detection (SUD) where each destination treats the interferences as Gaussian noise. Therefore, the signal-to-interference-plus-noise ratio (SINR) at the kth destination is given by

$$\text{SINR}_k = \frac{p_k|\mathbf{b}_k^H\mathbf{w}|^2}{||\mathbf{A}_k\mathbf{w}||^2 + ||\mathbf{B}_k\mathbf{w}||^2 + \sigma_d^2} \tag{5}$$

and the achievable rate of the kth destination is given by $R_k(\mathbf{w}) = \frac{1}{2}\log_2(1 + \text{SINR}_k)$.

Depending on the application, different design criteria will be considered. In this paper, we consider two typical performance measures:

- Achievable rate region;
- Sum-rate: $\frac{1}{2}\sum_{k=1}^K \log_2(1 + \text{SINR}_k)$.

We define the achievable rate region for two-hop networks as the set of rate-tuples for all destinations that can be simultaneously achievable under the given relay beamfroming vector $\mathbf{w}$ that satisfies the relay sum-power constraint (4):

$$\mathcal{R} \triangleq \bigcup_{\{\mathbf{w}:\, \mathbf{w}^H\mathbf{D}\mathbf{w}\le P\}} \{(r_1, ..., r_K) : 0 \le r_k \le R_k(\mathbf{w})\}. \tag{6}$$

The outer boundary of this region is called the *Pareto boundary*, because it consists of operating points $(R_1, ..., R_K)$ for which it is impossible to improve one of the rates, without simultaneously decreasing at least one of the other rates [3].

III. Optimal Coordinated Beamforming

In this section, we design the optimal beamforming matrices for cooperative relays to characterize the achievable rate region and maximize the sum-rate. We also propose a low-complexity iterative suboptimal algorithm to maximize the sum-rate.

A. Optimal Achievable Rate Region Characterization

The rate-profile method [3] is a well known technique to characterize the achievable rate region. Specifically, for a given rate-profile vector, $\boldsymbol{\nu} = (\nu_1, ..., \nu_K)$, we can solve the following optimization problem to obtain the corresponding rate-tuple on the *Pareto boundary* of the achievable rate region:

$$\begin{aligned} \max_{R_{\text{sum}},\mathbf{w}} \quad & R_{\text{sum}} \\ \text{s. t.} \quad & \frac{1}{2}\log_2(1 + \text{SINR}_k) \ge \nu_k R_{\text{sum}}, \quad k = 1, ..., K \\ & \mathbf{w}^H\mathbf{D}\mathbf{w} \le P. \end{aligned} \tag{7}$$

where $\boldsymbol{\nu}$ satisfies that $\nu_i \ge 0$, $1 \le i \le K$, and $\Sigma_{i=1}^K \nu_i = 1$. We denote the optimal solution of Problem (7) as R_{sum}^*, and then $R_{\text{sum}}^* \cdot \boldsymbol{\nu}$ is the corresponding Pareto optimal rate-tuple, as shown in [3]. Although the Problem (7) is nonconvex, we can use the bisection search algorithm to find R_{sum}^* as shown in [3]. Specifically, we could solve a sequence of relay sum-power minimization problems for a given rate-profile vector $\boldsymbol{\nu}$ and $R_{\text{sum}} = r_{\text{sum}}$:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H\mathbf{D}\mathbf{w} \\ \text{s. t.} \quad & \frac{p_k|\mathbf{b}_k^H\mathbf{w}|^2}{||\mathbf{A}_k\mathbf{w}||^2 + ||\mathbf{B}_k\mathbf{w}||^2 + \sigma_d^2} \ge \gamma_k, \ k = 1, ..., K, \end{aligned} \tag{8}$$

where $\gamma_k \triangleq 2^{2\nu_k r_{\text{sum}}} - 1$. Denote the optimal value of the above problem as p^*, which is the minimum sum-power required by the relays to support the target SINRs $\{\gamma_k\}_{k=1}^K$. Therefore, if $p^* \le P$, it follows that $\{\gamma_k\}_{k=1}^K$ are achievable and $R_{\text{sum}}^* \ge r_{\text{sum}}$; otherwise, $\{\gamma_k\}_{k=1}^K$ are not achievable and $R_{\text{sum}}^* < r_{\text{sum}}$. However, Problem (8) is still nonconvex.

Although Fadel *et al.* [5] proposed an algorithm to find the global optimal solution to Problem (8), it still has high complexity since it needs to solve a group of second-order cone programing (SOCP) problems. In order to increase the speed of testing if a specified set of SINR values are achievable, we propose to first bound the optimal value p^* as $p_L \le p^* \le p_U$, where p_L is the optimal value of the following semi-definite programming (SDP) problem:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \text{tr}(\mathbf{D}\mathbf{X}) \\ \text{s. t.} \quad & \text{tr}(\mathbf{J}_k\mathbf{X}) \ge \gamma_k\sigma_d^2, \quad \mathbf{X} \succeq 0, \quad k = 1, ..., K, \end{aligned} \tag{9}$$

where $\mathbf{X} = \mathbf{w}\mathbf{w}^H$ and $\mathbf{J}_k = p_k\mathbf{b}_k\mathbf{b}_k^H - \gamma_k(\mathbf{Q}_k + \mathbf{G}_k)$ and p_U is the optimal value of the following SOCP problem:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H\mathbf{D}\mathbf{w} \\ \text{s. t.} \quad & ||\mathbf{U}_k\mathbf{w} + \mathbf{v}_k|| \le \Re\{\mathbf{b}_k^H\mathbf{w}\}, \ k = 1, ..., K, \end{aligned} \tag{10}$$

where

$$\mathbf{U}_k \triangleq \sqrt{\frac{\gamma_k}{p_k}} \begin{bmatrix} \mathbf{A}_k \\ \mathbf{B}_k \\ \mathbf{0}_{\sum_{l=1}^L M_l^2}^T \end{bmatrix}, \mathbf{v}_k \triangleq \begin{bmatrix} \mathbf{0}_{K-1+\sum_{l=1}^L M_l} \\ \sqrt{\frac{\gamma_k}{p_k}}\sigma_d \end{bmatrix}.$$

The solution of (9) is a lower bound of p^* as we remove the rank-one constraint for $\mathbf{X}$; while the solution of (10) is an upper bound of p^* as we use the conservative approximation $|\mathbf{b}_k^H\mathbf{w}| \ge \Re\{\mathbf{b}_k^H\mathbf{w}\}$. Therefore, we can easily obtain p_L and p_U through solving the convex problems (9) and (10), respectively. If $p_U \le P$, then the target $\{\gamma_k\}_{k=1}^K$ are achievable. If $p_L \ge P$, then $\{\gamma_k\}_{k=1}^K$ are not achievable. If neither $p_U \le P$ nor $p_L \ge P$, we resort to the more computationally expensive algorithm [5] to obtain p^*. If $p^* \le P$, then $\{\gamma_k\}_{k=1}^K$ are achievable; otherwise, they are not achievable. Based on this observation, our proposed achievable rate region characterization (ARR) algorithm is presented at the top of the next page.

One way to determine the upper bound $r_{\max}$ in Algorithm 1 is as follows. Based on the Cauchy-Schwartz inequality, we can obtain $|\mathbf{b}_k^H\mathbf{w}|^2 \le ||\mathbf{b}_k^H||^2||\mathbf{w}||^2$ and $\mathbf{w}^H\mathbf{D}\mathbf{w} \le ||\mathbf{w}||^2\sum_{t=1}^T ||\mathbf{d}_t^H||^2$, where $\mathbf{D}^{1/2} = [\mathbf{d}_1, ..., \mathbf{d}_T]^H$ and $T = \sum_{l=1}^L M_l^2$. Using (4) and neglecting the interference and

Algorithm 1: Optimal Beamforming for ARR

Step 0: Given $R_{\text{sum}} \in [0, r_{\max}]$, $\boldsymbol{\nu}$.
Step 1: Initialize $r_{\text{low}} = 0$, $r_{\text{up}} = r_{\max}$.
Step 2: Repeat

1) Set $r \leftarrow \frac{1}{2}(r_{\text{low}} + r_{\text{up}})$;
2) Obtain p_U with the given r: if $p_U \le P$, set $r_{\text{low}} \leftarrow r$, **go to step 3**; otherwise, obtain p_L with the given r: if $p_L \ge P$, set $r_{\text{up}} \leftarrow r$, **go to step 3**; otherwise, obtain p^* with the given r;
3) Update r by the bisection method: if $p^* \le P$, set $r_{\text{low}} \leftarrow r$; otherwise, $r_{\text{up}} \leftarrow r$.

Step 3: Until $r_{\text{up}} - r_{\text{low}} < \delta$, where δ represents an accuracy requirement. The converged value of r_{up} is the optimal solution of $R_{\max}$ in (7).

forwarded noise terms in the denominator of SINR in (5), we can obtain an upper bound of SINR_k as $\gamma_{k,\max} = p_k||\mathbf{b}_k^H||^2 P/(\sigma_d^2 \sum_{t=1}^T ||\mathbf{d}_t^H||^2)$. Therefore, $r_{\max}$ can be given by $r_{\max} = 1/2 \sum_{k=1}^K \log(1+\gamma_{k,\max})$.

B. Sum-Rate Maximization

1) Optimal sum-rate maximization beamforming: Recently, Joshi *et al.* [4] proposed an algorithm based on the branch and bound technique to solve the weighted sum-rate maximization problem for MISO downlink cellular networks. This method can be easily extended to our cooperative relay networks. Due to the scope of the paper, we omit the details of the algorithm, which can be found in [4]. The main difference between our problem and the problem considered in [4] is testing if a specified set of SINR values $\{\gamma_k\}_{k=1}^K$ [4, (24)] are achievable. To reduce the computational complexity, we can follow exactly the same steps described in Section III. A to check if these SINR values are achievable. However, as the original problem is NP-hard, the optimal solution has a prohibitive complexity for practical implementation.

2) Suboptimal beamforming strategy to the sum-rate maximization: From the last subsection, we see that the global optimal solution has high complexity, but it can be used to provide performance benchmarks for designing low complexity algorithms. In this subsection, we propose a low complexity algorithm to design the beamforming matrices. From problem (8), we find that the main difficulty is that the equivalent effective scalar channel gains $h_k^{\text{signal}} = \mathbf{b}_k^H\mathbf{w},\ k=1,...,K$ are coupled with each other via the cooperative beamforming vector $\mathbf{w}$, which means it is impossible to use the phase rotation technique as in [12]. This is a fundamental difference from the single-hop MISO interference channel, for which we can use the phase rotation technique to make all the channel gains real and nonnegative at the same time [4, (3)] without change the objective and the constraints. To overcome the difficulty, we propose a method to decouple the equivalent effective channels, i.e., adding the following constraint:

$$\mathbf{b}_k^H\mathbf{w} = \mathbf{b}_{k'}^H\mathbf{w}, \quad \forall k, k' = 1,...,K. \tag{11}$$

After aligning these channel gains at the same level, relays can jointly design the beamforming matrices to maximize the sum-rate in a more tractable way. Let us introduce the $(K-1) \times \sum_{l=1}^L M_l^2$ matrix $\bar{\mathbf{F}} \triangleq \left[(\mathbf{b}_1^H - \mathbf{b}_2^H)^T, ..., (\mathbf{b}_1^H - \mathbf{b}_K^H)^T\right]^T$. We will assume that $\sum_{l=1}^L M_l^2 > (K-1)$ and $\bar{\mathbf{F}}$ is full row-rank for independent channels. Thus, the constraint (11) can be rewritten as $\bar{\mathbf{F}}\mathbf{w} = \mathbf{0}$. The nontrivial solution to it is given by

$$\bar{\mathbf{w}} = \bar{\mathbf{U}}\boldsymbol{\alpha}, \tag{12}$$

where $\boldsymbol{\alpha}$ is an arbitrary $[\sum_{l=1}^L M_l^2 - (K-1)] \times 1$ vector, $\bar{\mathbf{U}} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, ..., \boldsymbol{\varphi}_{\sum_{l=1}^L M_l^2-(K-1)}]$ and $\boldsymbol{\varphi}_i$ is the eigenvector corresponding to the ith zero eigenvalue of $\bar{\mathbf{F}}^H\bar{\mathbf{F}}$. Thus, the SINR at the kth destination is given by

$$\overline{\text{SINR}}_k = \frac{p_k|\mathbf{f}^H\boldsymbol{\alpha}|^2}{\boldsymbol{\alpha}^H\bar{\mathbf{R}}_k\boldsymbol{\alpha} + \sigma_d^2}, \tag{13}$$

where $\mathbf{f}^H \triangleq \mathbf{b}_1^H\bar{\mathbf{U}}$ and $\bar{\mathbf{R}}_k \triangleq \bar{\mathbf{U}}^H(\mathbf{Q}_k + \mathbf{G}_k)\bar{\mathbf{U}}$. The sum-rate maximization problem becomes more tractable based on this form of SINR. First, let us introduce the following lemma.

Lemma 1: [13] For any positive real numbers $f_1, f_2, ..., f_K$ we have:

$$\min_{\mathbf{t}\in\mathcal{T}} \left(\frac{1}{K}\sum_{k=1}^K f_k t_k\right)^K = \prod_{k=1}^K f_k, \tag{14}$$

where $\mathcal{T} \triangleq \{\mathbf{t} \in \mathbb{R}_+^k : \prod_{k=1}^K t_k = 1\}$.

Then, consider the following sum-rate maximization problem, i.e., to maximize the sum-rate after decoupling:

$$\max_{\boldsymbol{\alpha}} \ \frac{1}{2}\sum_{k=1}^K \log_2(1+\overline{\text{SINR}}_k) \ \text{s. t.}\ \boldsymbol{\alpha}^H\bar{\mathbf{D}}\boldsymbol{\alpha} \le P, \tag{15}$$

where $\bar{\mathbf{D}} \triangleq \bar{\mathbf{U}}^H\mathbf{D}\bar{\mathbf{U}}$. At high SNR, this problem can be approximated to

$$\min_{\boldsymbol{\alpha}} \ \prod_{k=1}^K \frac{1}{\overline{\text{SINR}}_k} \quad \text{s. t.}\ \ \boldsymbol{\alpha}^H\bar{\mathbf{D}}\boldsymbol{\alpha} \le P. \tag{16}$$

Based on Lemma 1 and (16), the suboptimal $\boldsymbol{\alpha}$ of (15) can be obtained by solving the following problem

$$\min_{\boldsymbol{\alpha},\mathbf{t}\in\mathcal{T}} \ \sum_{k=1}^K \frac{t_k}{\overline{\text{SINR}}_k} \quad \text{s. t.}\ \ \boldsymbol{\alpha}^H\bar{\mathbf{D}}\boldsymbol{\alpha} \le P. \tag{17}$$

This problem can be solved iteratively. Specifically, for a fixed $\boldsymbol{\alpha}$, the optimal $\mathbf{t}$ of (17) are given by [13, (12)]

$$t_k = \frac{\overline{\text{SINR}}_k}{\left(\prod_{k=1}^K \overline{\text{SINR}}_k\right)^{1/K}}, \quad \forall k = 1,...,K. \tag{18}$$

For a fixed $\mathbf{t}$, the optimization problem (17) is given by

$$\min_{\boldsymbol{\alpha}} \ \sum_{k=1}^K t_k \frac{\boldsymbol{\alpha}^H\bar{\mathbf{R}}_k\boldsymbol{\alpha} + \sigma_d^2}{p_k\boldsymbol{\alpha}^H\mathbf{f}\mathbf{f}^H\boldsymbol{\alpha}} \quad \text{s. t.}\ \ \boldsymbol{\alpha}^H\bar{\mathbf{D}}\boldsymbol{\alpha} \le P. \tag{19}$$

It is obvious that the optimal $\boldsymbol{\alpha}^*$ must satisfy $(\boldsymbol{\alpha}^*)^H\bar{\mathbf{D}}\boldsymbol{\alpha}^* = P$. Thus, the above problem is equivalent to

$$\boldsymbol{\alpha}^* = \arg\max \frac{\boldsymbol{\alpha}^H\mathbf{f}\mathbf{f}^H\boldsymbol{\alpha}}{\boldsymbol{\alpha}^H\mathbf{R}\boldsymbol{\alpha}}, \tag{20}$$

where $\mathbf{R} = \sum_{k=1}^{K} (\frac{t_k}{p_k}\overline{\mathbf{R}}_k + \frac{t_k \sigma_d^2}{p_k P}\bar{\mathbf{D}})$. It can be written in the form of the Rayleigh-Ritz ratio and the optimal solution is thus given as

$$\boldsymbol{\alpha}^* = \xi \mathbf{R}^{-1}\mathbf{f}, \quad (21)$$

where $\xi > 0$ is a scalar such that $(\boldsymbol{\alpha}^*)^H \bar{\mathbf{D}} \boldsymbol{\alpha}^* = P$. Thus, our proposed iterative algorithm that provides a suboptimal solution to (15) is summarized as follows.

Algorithm 2: Iterative Algorithm for Sum-Rate Maximization

Step 0: Initialize $t_1 = t_2 = \cdots = t_K = 1$, and obtain $\bar{\mathbf{U}}$ through eigenvalue decomposition of $\bar{\mathbf{F}}^H \bar{\mathbf{F}}$.
Step 1: Repeat
1) Update $\boldsymbol{\alpha}^*$ using (21) with a fixed $\mathbf{t}$,
2) Update $\{t_k\}_{k=1}^{K}$ using (18) with a fixed $\boldsymbol{\alpha}$.
Step 2: Until convergence.
Step 3: Obtain $\bar{\mathbf{w}}$ using (12).

Convergence: At each step, the objective function of (17) is non-increasing and the global optimum of it is also achieved. The facts show that the iterative algorithm is guaranteed to converge to the local optimum of problem (17).

IV. A Suboptimal Solution: Interference Neutralization Beamforming

In this section, we present a new cooperative interference management scheme, termed *interference neutralization*, which allows interference to be canceled over the air at the last hop. Here, the relay beamformer $\mathbf{w}$ is designed to neutralize interferences at each destination terminal, i.e.,

$$\tilde{\mathbf{F}}\mathbf{w} = \mathbf{0}, \quad (22)$$

where $\tilde{\mathbf{F}} \triangleq [\mathbf{f}_{1,2}^T, ..., \mathbf{f}_{k,k'}^T, ..., \mathbf{f}_{K,K-1}^T]^T$, for $k \neq k'$, is the $K(K-1) \times \sum_{l=1}^{L} M_l^2$ matrix with the $1 \times \sum_{l=1}^{L} M_l^2$ vector $\mathbf{f}_{k,k'} \triangleq [\mathbf{h}_{1,k'}^T \otimes \mathbf{g}_{k,1}^T, ..., \mathbf{h}_{L,k'}^T \otimes \mathbf{g}_{k,L}^T]$. We assume that $\sum_{l=1}^{L} M_l^2 > K(K-1)$ and $\tilde{\mathbf{F}}$ is full row-rank. A nontrivial solution to (22) is given by any linear combination of the eigenvectors corresponding to the null space of $\tilde{\mathbf{F}}^H \tilde{\mathbf{F}}$, i.e., $\tilde{\mathbf{w}} = \tilde{\mathbf{U}}\boldsymbol{\beta}$, where $\boldsymbol{\beta}$ is an arbitrary $[\sum_{l=1}^{L} M_l^2 - K(K-1)] \times 1$ vector, $\tilde{\mathbf{U}} = [\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_{\sum_{l=1}^{L} M_l^2 - K(K-1)}]$ and $\mathbf{u}_i$ are the eigenvector corresponding to the ith zero eigenvalue of $\tilde{\mathbf{F}}^H \tilde{\mathbf{F}}$. As a result, the SINR at the kth destination is given by

$$\widehat{\mathrm{SINR}}_k = \frac{p_k |\tilde{\mathbf{b}}_k^H \boldsymbol{\beta}|^2}{||\tilde{\mathbf{B}}_k \boldsymbol{\beta}||_2^2 + \sigma_d^2}, \quad (23)$$

where $\tilde{\mathbf{b}}_k^H \triangleq \mathbf{b}_k^H \tilde{\mathbf{U}}$ and $\tilde{\mathbf{B}}_k \triangleq \mathbf{B}_k \tilde{\mathbf{U}}$.

A. Achievable Rate Region Characterization

To characterize the achievable rate region for the interference neutralization scheme, we only need to determine the $[\sum_{l=1}^{L} M_l^2 - K(K-1)] \times 1$ vector $\boldsymbol{\beta}$. Whereas, we need to determine the $\sum_{l=1}^{L} M_l^2 \times 1$ beamforming vector $\mathbf{w}$ for the optimal coordinated beamforming. We can use the same steps described in section III. A to characterize the achievable rate region for the interference neutralization scheme, letting $\mathbf{0} \rightarrow \mathbf{A}_k$, $\tilde{\mathbf{B}}_k \rightarrow \mathbf{B}_k$, $\tilde{\mathbf{D}} \rightarrow \mathbf{D}$, $\tilde{\mathbf{b}}_k^H \rightarrow \mathbf{b}_k^H$ and $\boldsymbol{\beta} \rightarrow \mathbf{w}$.

B. Sum-Rate Maximization

In principle, we can also use the same algorithm described in Section III. B 1) to find the global optimal solution for the sum-rate maximization problem. However, to avoid the high complexity of that algorithm, in this section, we propose a suboptimal solution for the interference neutralization scheme. Besides the interference neutralization constraint on the relay beamforming vector $\mathbf{w}$, we impose a linear constraint on the desired signals to preserve the signal level, i.e.,

$$[\mathbf{b}_1^*, ..., \mathbf{b}_K^*]^T \mathbf{w} = \zeta \mathbf{1}_K. \quad (24)$$

By combining the constraints (22) and (24), we can rewrite the two constraints as $\mathbf{F}\mathbf{w} = \zeta\mathbf{c}$, where $\mathbf{F} \triangleq [\tilde{\mathbf{F}}; [\mathbf{b}_1^*, ..., \mathbf{b}_K^*]^T]$, $\mathbf{c} \triangleq [\mathbf{0}_{K(K-1)}; \mathbf{1}_K]$, and $\zeta \in \mathbb{C}$ is selected to meet the sum-power constraint at relays. Thus, we need to solve the following optimization problem

$$\min_{\mathbf{w}} \ \mathbf{w}^H \mathbf{D} \mathbf{w} \ \ \text{s. t.} \ \ \mathbf{F}\mathbf{w} = \mathbf{c}. \quad (25)$$

This is a convex optimization problem. Using the Lagrange multipliers method, the optimal solution to it can be given by $\check{\mathbf{w}} = \mathbf{D}^{-1}\mathbf{F}^H[\mathbf{F}\mathbf{D}^{-1}\mathbf{F}^H]^{-1}\mathbf{c}$. Therefore, the suboptimal solution for the interference neutralization scheme is given in the closed-form as

$$\hat{\mathbf{w}} = \zeta \check{\mathbf{w}}, \quad (26)$$

where $\zeta = \sqrt{P_R / (\check{\mathbf{w}}^H \mathbf{D} \check{\mathbf{w}})}$ is to satisfy the relay sum-power constraint $\hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} = P$.

V. Numerical Results

In this section, numerical results are presented to compare the performance of different coordinated relay beamforming methods in terms of the achievable rate region and the achievable sum-rate.

In Fig. 2, we consider a scenario with $K = 2$ source-destination pairs communicating through $L = 2$ relays with 2 antennas each. In our simulation, it is assumed that all sources and relays have the same transmit power p and $\sigma_r^2 = \sigma_d^2 = 1$, and the SNR is defined as p/σ_r^2, which is set to be 10 dB. All channel coefficients are generated from independent circularly symmetric complex Gaussian variables with distribution $\mathcal{CN}(0, 1)$. Each rate pair point is based on averaging over 100 independent and random channel realizations. This figure shows that the interference neutralization scheme provides good performance with a relatively low complexity compared with the optimal coordinated relay beamforming.

In Fig. 3, we consider a scenario of $K = 3$ source-destination pairs and $L = 3$ 2-antenna relays assisting the data transmission. We assume that all sources have the same transmit power p and all relays have the same transmit power $0.8p$ and $\sigma_r^2 = \sigma_d^2 = 1$, and the SNR is defined as p/σ_r^2. All direct-link channel coefficients $\mathbf{h}_{i,i}$ and $\mathbf{g}_{i,i}$ are generated from i.i.d. $\mathcal{CN}(0,1)$ and all cross-link channel coefficients $\mathbf{h}_{i,j}$ and $\mathbf{g}_{i,j}$ are generated from i.i.d. $\mathcal{CN}(0,1)$ and multiplied by a factor $1/2$ to indicate different path losses. For comparison, the total leakage minimization algorithms subject to signal preservation constraints [6] or a relay sum-power constraint [7] are also evaluated. These are denoted by "Min leakage algorithm 1" and "Min leakage algorithm 2", respectively. This figure shows that our proposed Algorithm 2 achieves the best performance among the evaluated relay beamforming algorithms. Moreover, the proposed closed-form solution (26) for the interference neutralization scheme also outperforms "Min leakage algorithm 1" in the high SNR region. Furthermore, at the medium to high SNRs, all the cooperative relay beamforming strategies outperform the non-cooperative relay beamforming scheme [14] where each relay only knows its direct-link channel coefficients.

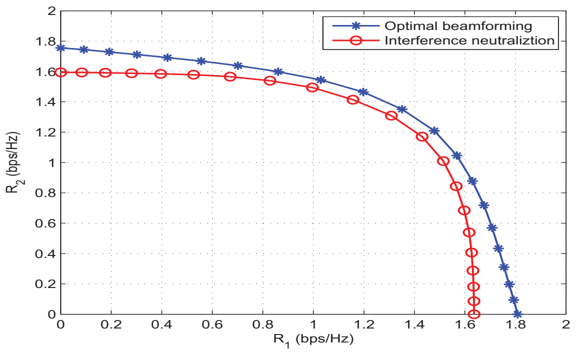

Fig. 2. Achievable rate region at SNR=10 dB.

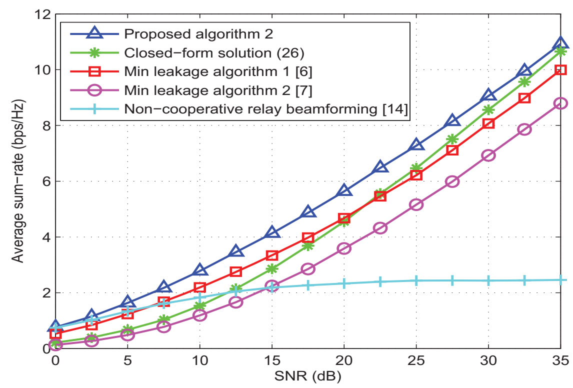

Fig. 3. Achievable sum-rates of different relay beamforming methods.

In Table I, we choose one channel realization [1] from the same scenario considered in Fig. 3 to compare the proposed low-complexity Algorithm 2 with the optimal beamforming. It is observed that our proposed algorithm 2 only has a performance loss about 10% compared with the optimal relay beamforming, but it has very low complexity and can be implemented in real-time.

[1]The corresponding channel coefficients for the realization can be found at http://www.ece.ust.hk/~eejzhang/document/channel.mat

TABLE I
SUM-RATE COMPARISONS FOR DIFFERENT BEAMFORMING STRATEGIES

SNR (dB)	5	10	15	20	25
Optimal beamforming	2.35	4.07	6.08	8.37	10.73
Proposed Algorithm 2	1.99	3.56	5.43	7.53	9.85

VI. CONCLUSION

In this paper, we investigated coordinated relay beamforming for interference mitigation in two-hop interference networks. We first provided performance benchmarks by developing optimal coordinated relay beamforming algorithms. It was demonstrated that the proposed iterative Algorithm 2 always outperforms exiting suboptimal relay beamforming algorithms. We also showed that the proposed interference neutralization scheme can provide good performance with a relatively low complexity, especially at high SNR. Possible future research directions include considering a general setting with arbitrary number of antennas at all terminals and developing practical CSI acquisition methods.

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