Scalable Sparse Optimization in Dense Cloud-RAN

Yuanming Shi

Supervisor: Khaled B. Letaief

Department of Electronic and Computer Engineering, HKUST

August 19, 2015

THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Outline

- **Introduction**
- **Three Vignettes:**
  - **Sparse optimization** for Green Cloud-RAN
  - **Chance Constrained Optimization** for Partially Connected Cloud-RAN
  - **Large-Scale Convex Optimization** for Dense Cloud-RAN
- **Summary**

Part I: Introduction

Ultra Mobile Broadband

- **Era of mobile data traffic deluge**

10x

Data growth
by 2019

497M

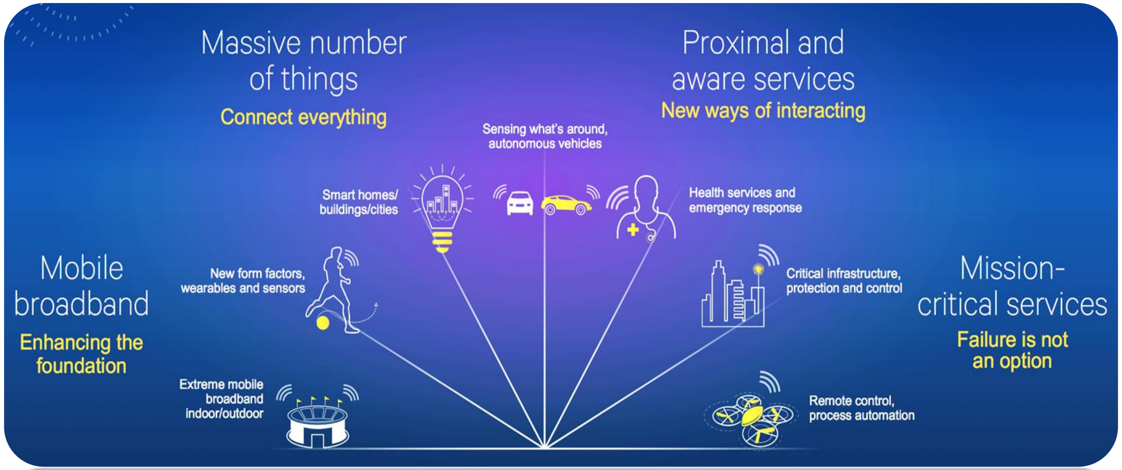
Mobile devices
added in 2014

72%

Video traffic by 2019

Source: Cisco VNI Mobile, 2015

We Need…

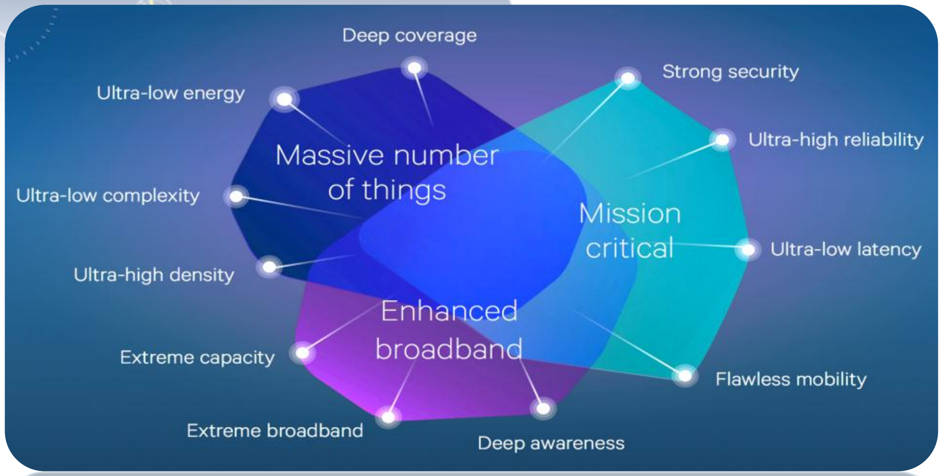


Support

current and emerging services

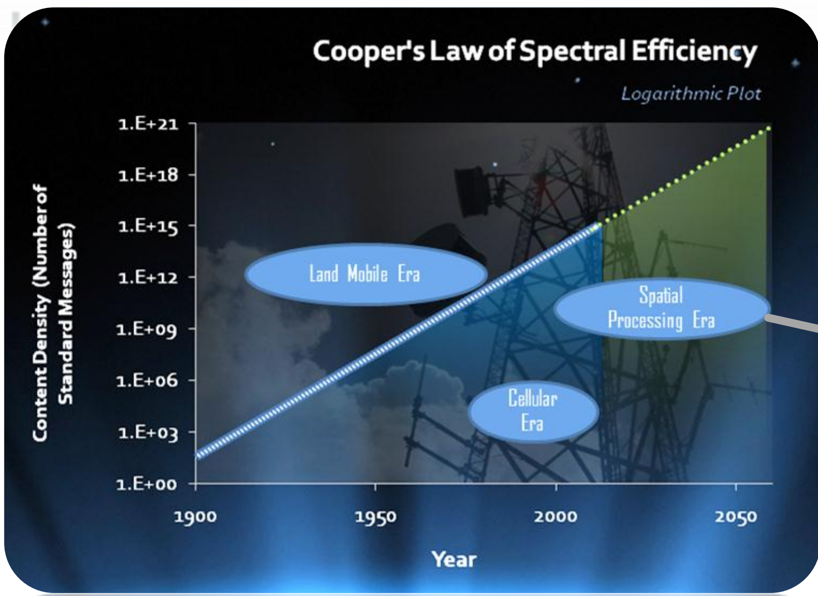
Scalable

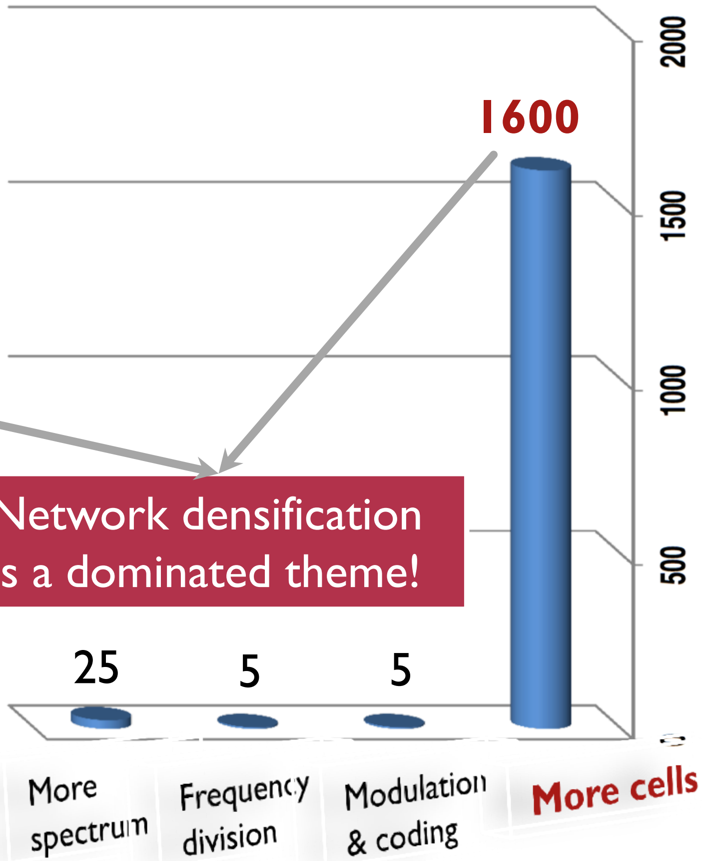
across an extreme variation



Solution?

Marty Pens Cooper's Law: Data Over Usable Spectrum Doubles Every 30 Months – 1997

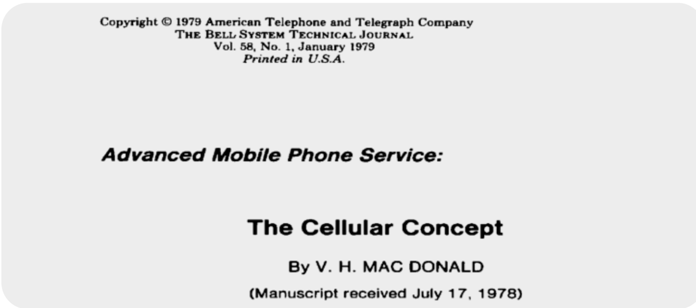




Network densification is a dominated theme!

Network Densification

- **Ultra-dense networking:** Coverage & capacity

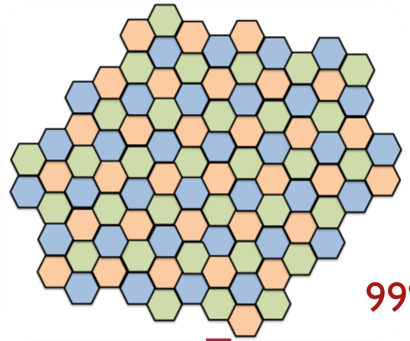

Copyright © 1979 American Telephone and Telegraph Company
THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 58, No. 1, January 1979
Printed in U.S.A.

Advanced Mobile Phone Service:

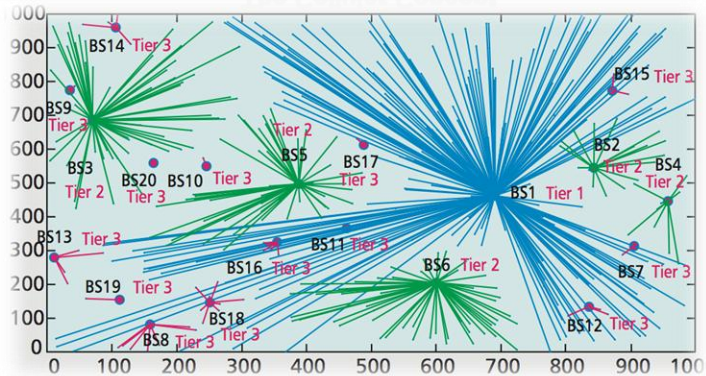
The Cellular Concept

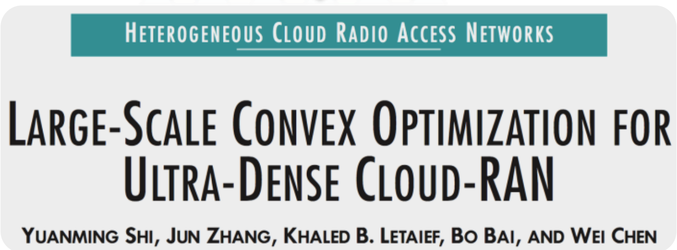
By V. H. MAC DONALD

(Manuscript received July 17, 1978)



99% coverage?




HETEROGENEOUS CLOUD RADIO ACCESS NETWORKS

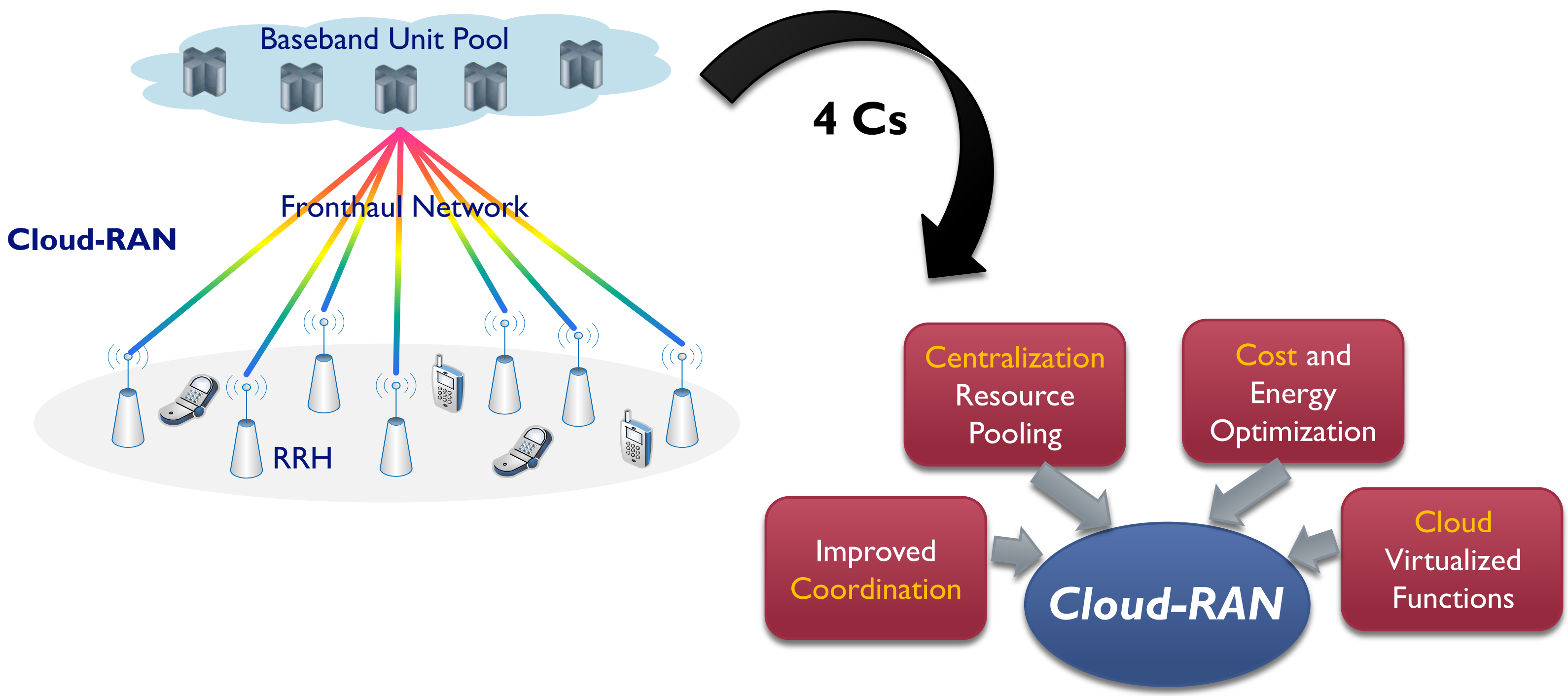
LARGE-SCALE CONVEX OPTIMIZATION FOR ULTRA-DENSE CLOUD-RAN

YUANMING SHI, JUN ZHANG, KHALED B. LETAIEF, BO BAI, AND WEI CHEN

Ultra-high capacity & uniform coverage

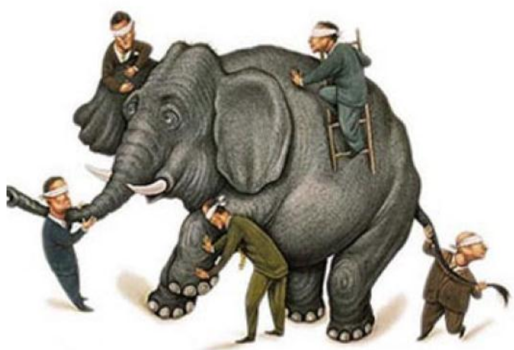
Dense Cloud Radio Access Networks

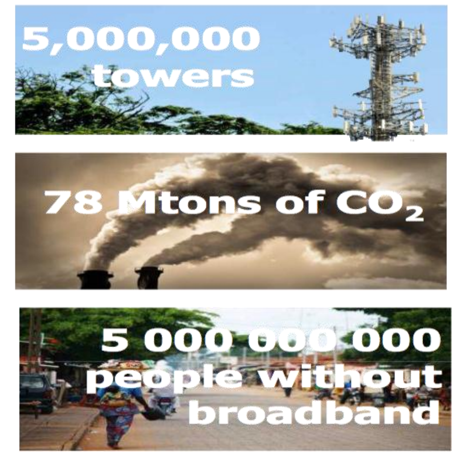
- **Dense Cloud-RAN:** A cost-effective way for network densification and cooperation



Challenges: Green, Flexibility, Scalability

- **Networking issues:**
  - Huge network power consumption
  - Massive channel state information acquisition



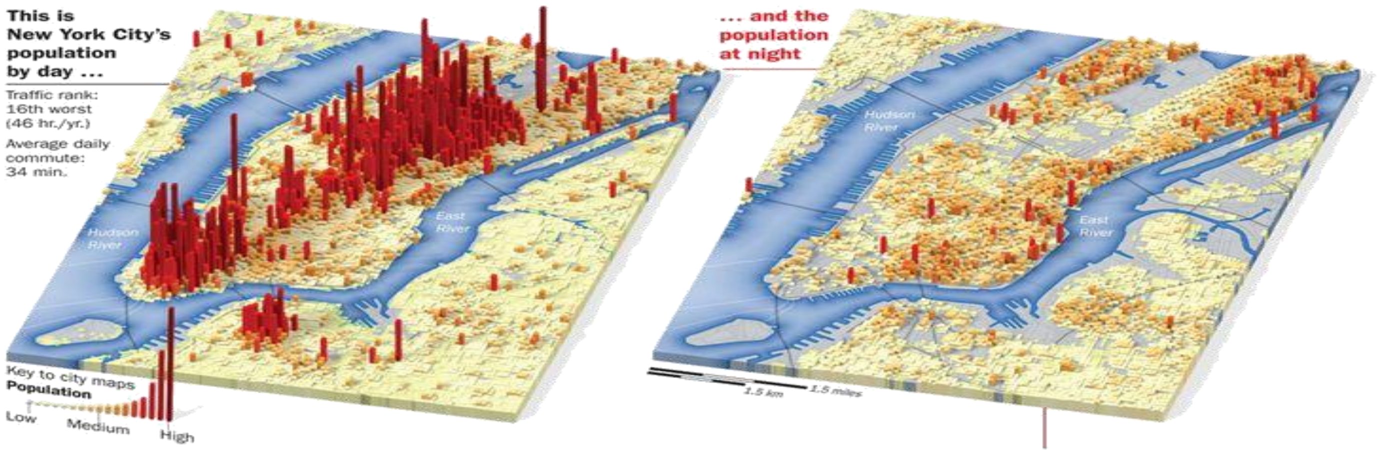


Source: Alcatel-Lucent, 2013

- **Computing issues:**
  - Large-scale performance optimizations
  - Limited computational resources

Networking Issues: Power Consumption

- **Group sparse optimization [1], [2]:** Network power minimization via network adaptation

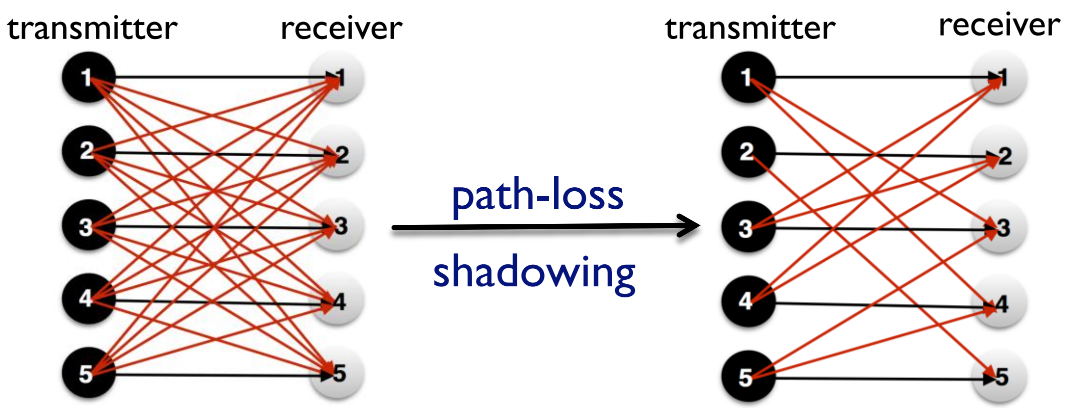


[1] Y. Shi, J. Zhang, and K. B. Letaief, "Group sparse beamforming for green Cloud-RAN," *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2809-2823, May 2014.

[2] Y. Shi, J. Zhang, and K. B. Letaief, "Robust group sparse beamforming for multicast green Cloud-RAN with imperfect CSI," *IEEE Trans. Signal Process.*, vol. 63, no. 17, pp. 4647-4659, Sept. 2015.

Networking Issues: Massive CSI

- **Low-rank matrix completion [3]:** Topological interference management
- **Sequential convex optimization [4]:** Stochastic coordinated beamforming

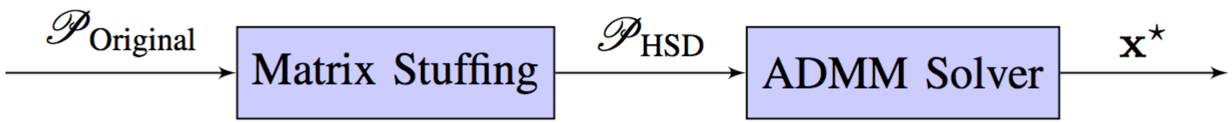


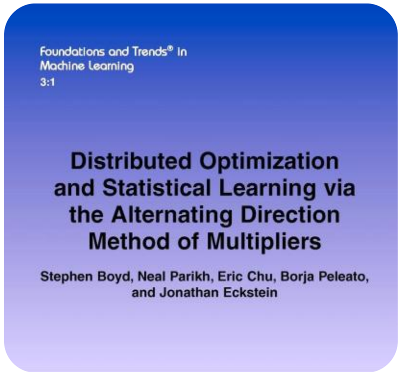
[3] Y. Shi, J. Zhang, and K. B. Letaief, "Low-rank matrix completion via Riemannian pursuit for topological interference management," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT),* Hong Kong, Jun. 2015.

[4] Y. Shi, J. Zhang, and K. B. Letaief, "Optimal stochastic coordinated beamforming for wireless cooperative networks with CSI uncertainty," *IEEE Trans. Signal Process.*, vol. 63, no. 4, pp. 960-973, Feb. 2015.

Computing Issues: Scalable Optimization

- **Two-stage large-scale convex optimization framework [5], [6]**



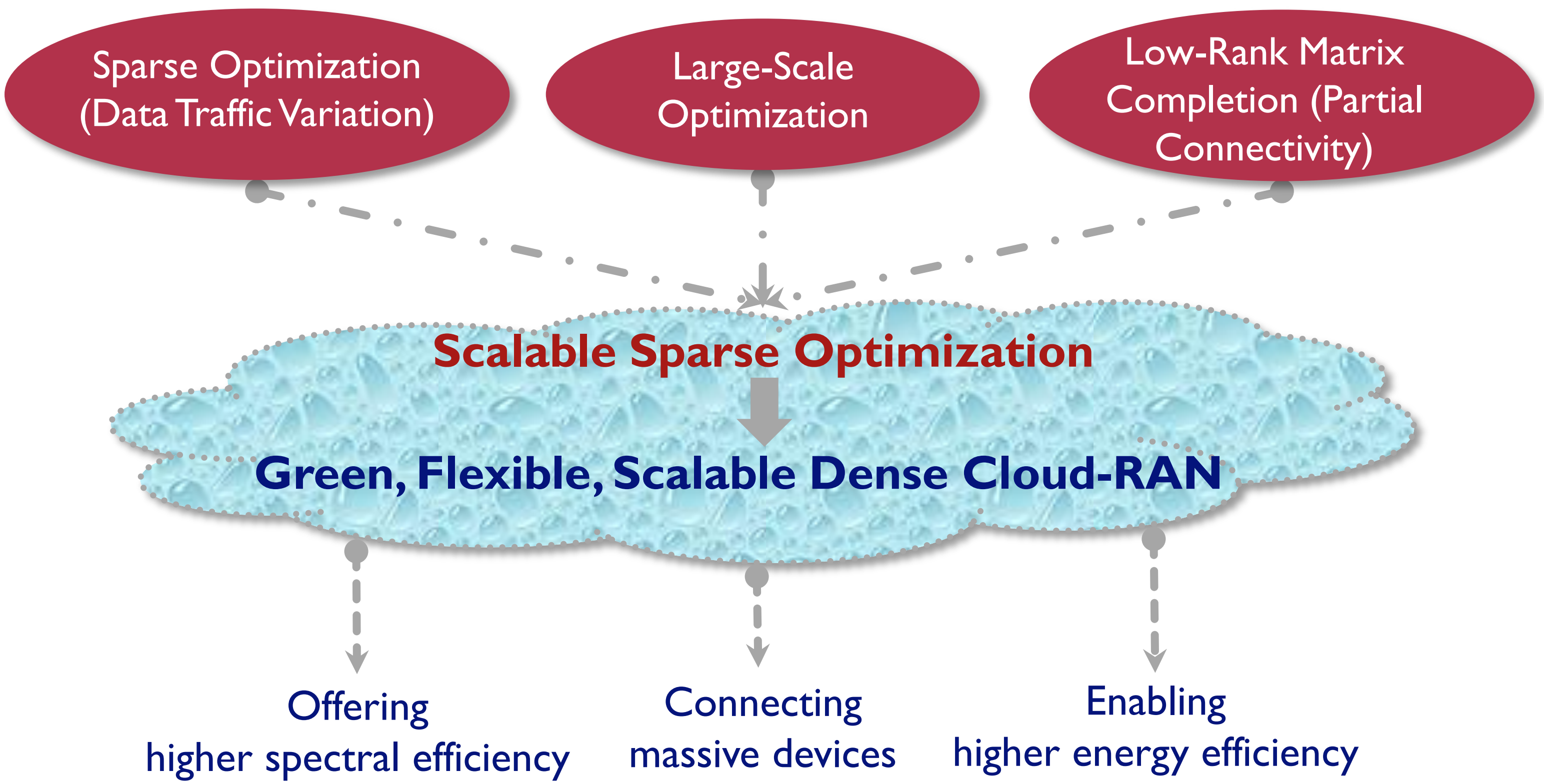


[5] Y. Shi, J. Zhang, K. B. Letaief, B. Bai and W. Chen,"Large-scale convex optimization for ultra-dense Cloud-RAN," *IEEE Wireless Commun. Mag*., pp. 84-91, Jun. 2015.

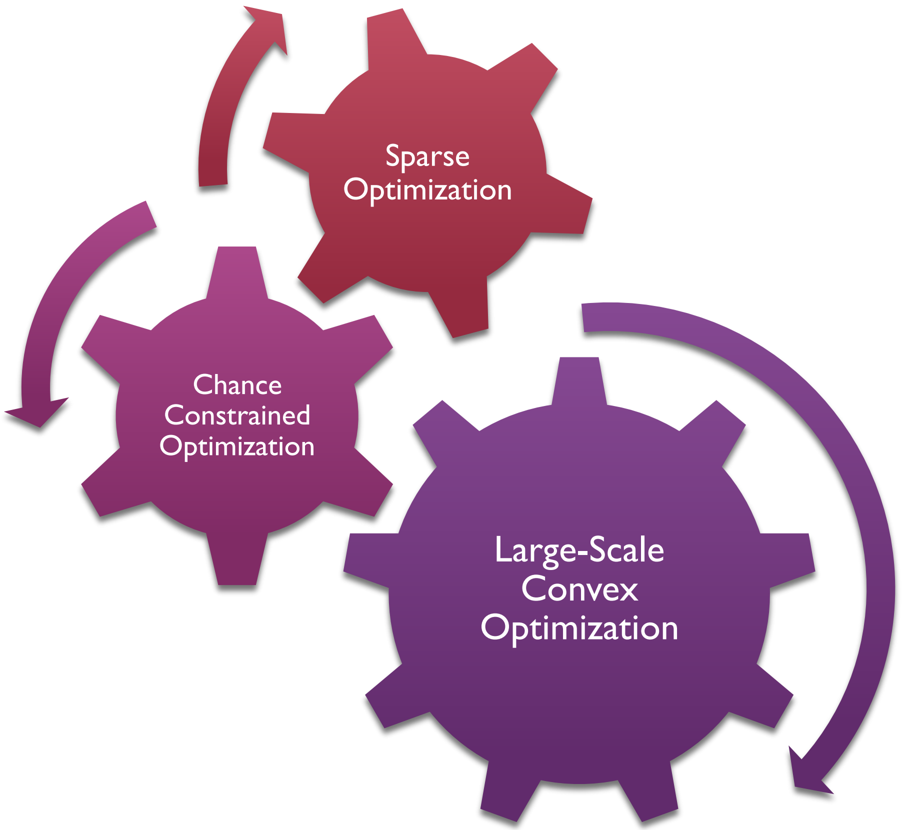
[6] Y. Shi, J. Zhang, B. O'Donoghue, and K. B. Letaief, "Large-scale convex optimization for dense wireless cooperative networks," *IEEE Trans. Signal Process*., vol. 63, no. 18, pp. 4729-4743, Sept. 2015.

Sparse Optimization for Dense Cloud-RAN

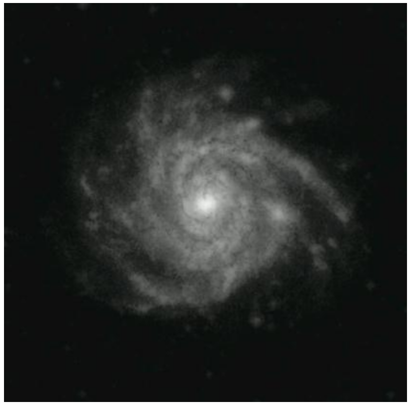
- **Findings:** 1) Dense network is well structured; 2) Sparse optimization is powerful to exploit such structures; 3) Scalable optimization is needed



Part II: Three Vignettes



Vignette A: Group Sparse Beamforming for Network Adaptation in Green Cloud-RAN

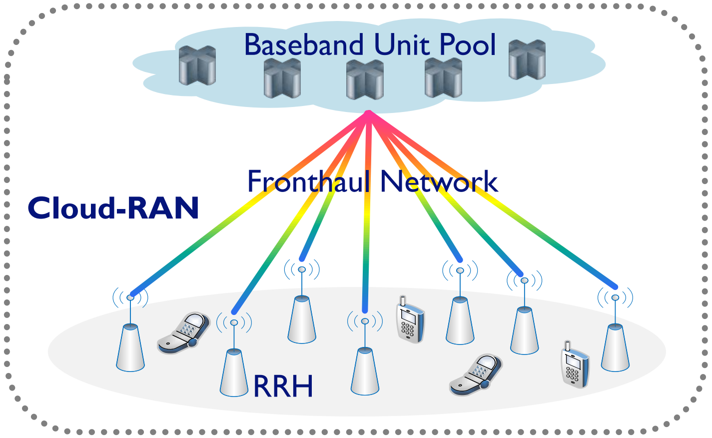


Issue A: Network Power Consumption

- **Goal:** Design a green dense Cloud-RAN

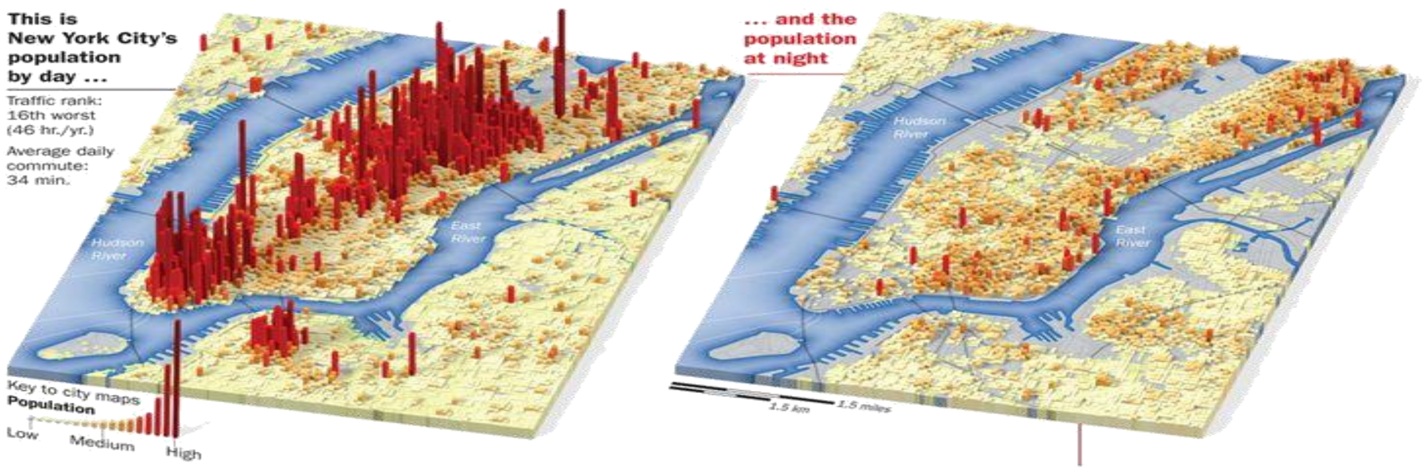
- **Prior works:** Physical-layer transmit power consumption
  - Wireless power control: [Chiang, *et al.*, FT 08], [Qian, *et al.*, TWC 09], [Sorooshyari, *et al.*, TON 12], …
  - Transmit beamforming: [Sidiropoulos and Luo, TSP 2006], [Yu and Lan, TSP 07], [Gershman, et al., SPMag 10],…

- **Unique challenge:**
  - Network power consumption:
    - RRHs, fronthaul links, etc.



Network Adaptation

- **Question:** Can we provide a holistic approach for network power minimization?
- **Key observation:** Spatial and temporal mobile data traffic variation



- **Approach:** Network adaptation
  - Switch off network entities to save power

Problem Formulation

- **Goal:** Minimize network power consumption in Cloud-RAN

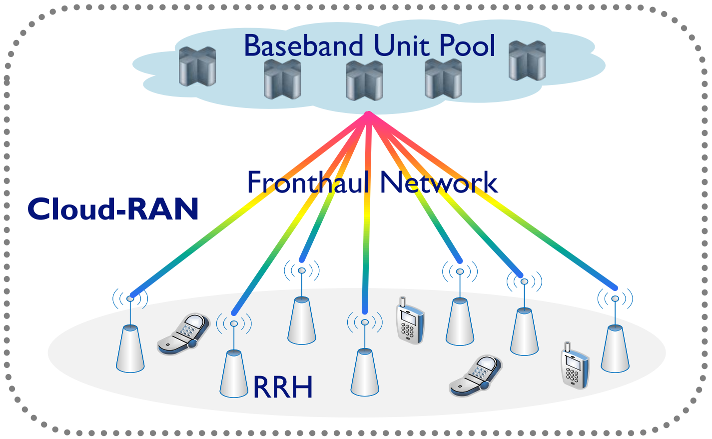
$$\underset{\mathbf{v}\in\mathcal{C}}{\text{minimize}}\ \underbrace{\sum_{l=1}^{L} P_l^c I(\mathrm{Supp}(\mathbf{v})\cap\mathcal{G}_l\neq\emptyset)}_{\text{fronthaul power}}+\underbrace{\sum_{l=1}^{L}\sum_{k=1}^{K}\frac{1}{\eta_l}\|\mathbf{v}_{lk}\|_2^2}_{\text{transmit power}}$$

NP-hard

- **Many applications:** Minimize a combinatorial composite function
  - Base station clustering [Hong, et al., JSAC 13], backhaul data assignment [Zhuang-Lau, TSP 13], user admission [Matskani, et al., TSP 09],…
- **Prior algorithms: Heuristic or computationally expensive:** [Philipp, et. al, TSP 13], [Luo, et. al, JSAC 13], [Quek, et. al, TWC 13],…

Finding Structured Solutions

- **Proposal:** Group sparse beamforming framework



$$\mathbf{v} = [\underbrace{\mathbf{v}_{11}^T, \ldots, \mathbf{v}_{1K}^T}_{\tilde{\mathbf{v}}_1^T}, \ldots, \underbrace{\mathbf{v}_{L1}^T, \ldots, \mathbf{v}_{LK}^T}_{\tilde{\mathbf{v}}_L^T}]^T$$

Beamforming coefficients of the first RRH, forming a **group**

- Switch off the l-th RRH $\rightarrow$ $\tilde{\mathbf{v}}_l = \mathbf{0}$, i.e., group sparsity structure in $\mathbf{v}$
- **Proposition [1]:** The *tightest* convex positively homogeneous lower bound of the combinatorial composite objective function

$$\Omega(\mathbf{v}) = 2\sum_{l=1}^{L}\sqrt{\frac{P_l^c}{\eta_l}}\|\tilde{\mathbf{v}}_l\|_2$$

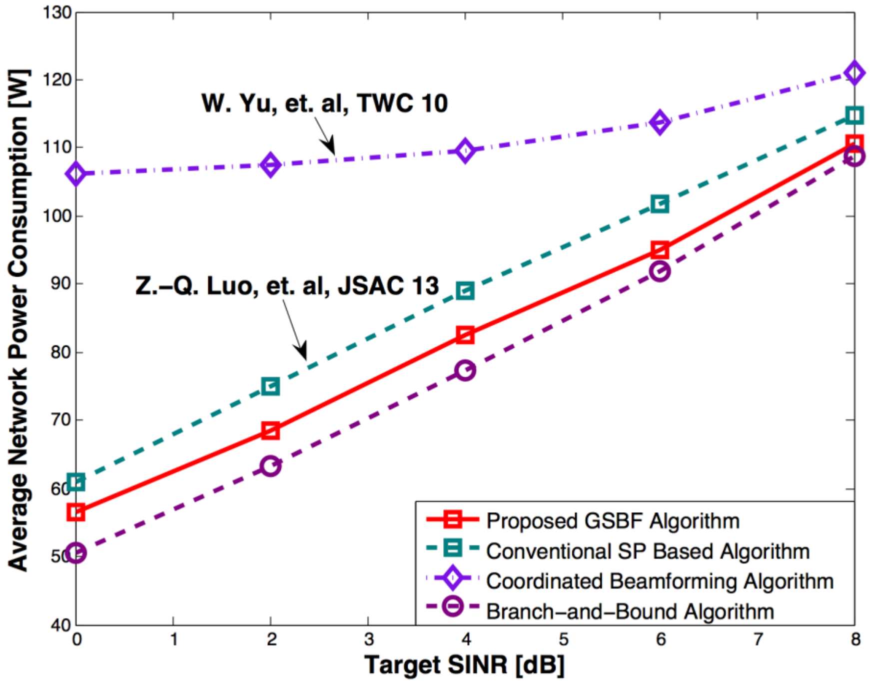
$$\underset{\mathbf{v}\in\mathcal{C}}{\text{minimize}} \ \Omega(\mathbf{v})$$

mixed ℓ_1/ℓ_2-norm

induce group sparsity

The Power of Group Sparse Beamforming

- **Example:** Group spare beamforming for green Cloud-RAN [1] (10 RRHs, 15 MUs)



Advantages:
1) Enabling flexible network adaptation;
2) Offering efficient algorithm design via convex programming
3) Empowering wide applications

[1] Y. Shi, J. Zhang, and K. B. Letaief, "Group sparse beamforming for green Cloud-RAN," *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2809-2823, May 2014.

Extensions: Multicast Cloud-RAN

- **Multi-group multicast transmission** in Cloud-RAN
  - All the users in the same group request the same message

$$\mathsf{SINR}_{k,m}(\mathbf{v})=\frac{\mathbf{v}_m^{\mathsf{H}}\mathbf{\Theta}_k\mathbf{v}_m}{\sum_{i\neq m}\mathbf{v}_i^{\mathsf{H}}\mathbf{\Theta}_k\mathbf{v}_i+\sigma_k^2}\geq\gamma_k,\forall k\in\mathcal{G}_m$$

- **Coupled challenges:**
  - Non-convex quadratic QoS constraints due to multicast transmission

$$F_{k,m}(\mathbf{v})=\gamma_k\left(\sum_{i\neq m}\mathbf{v}_i^{\mathsf{H}}\mathbf{\Theta}_k\mathbf{v}_i+\sigma_k^2\right)-\mathbf{v}_m^{\mathsf{H}}\mathbf{\Theta}_k\mathbf{v}_m\leq 0,\forall k\in\mathcal{G}_m$$

  - Combinatorial composite objective function: Network power consumption

$$P(\mathbf{v})=F(\mathrm{Supp}(\mathbf{v}))+T(\mathbf{v})$$

Multicast Group Sparse Beamforming

- **Semidefinite relaxation**: Convexify non-convex quadratic constraints
  - Lifting: $\mathbf{Q}_m = \mathbf{v}_m^{\mathsf{H}}\mathbf{v}_m \in \mathbb{C}^{N\times N}$

$$\gamma_k\left(\sum_{i\neq m}\mathbf{v}_i^{\mathsf{H}}\mathbf{\Theta}_k\mathbf{v}_i+\sigma_k^2\right)-\mathbf{v}_m^{\mathsf{H}}\mathbf{\Theta}_k\mathbf{v}_m := \gamma_k\left(\sum_{i\neq m}\mathrm{Tr}(\mathbf{\Theta}_k\mathbf{Q}_i)+\sigma_k^2\right)-\mathrm{Tr}(\mathbf{\Theta}_k\mathbf{Q}_m)$$

- **Quadratic variational formulation** of non-smooth mixed ℓ_1/ℓ_2-norm: Induce group sparsity in the multicast beamforming vector $\mathbf{v}$ [2]
  - Smoothing: $\mathbf{Q}_m = \mathbf{v}_m^{\mathsf{H}}\mathbf{v}_m \in \mathbb{C}^{N\times N}$

$$\Omega(\mathbf{v})^2=\left(\sum_{l=1}^{L}\sqrt{\omega_l}\|\tilde{\mathbf{v}}_l\|_2\right)^2 := \inf_{\boldsymbol{\mu}\in\mathcal{X}}\sum_{l=1}^{L}\frac{\omega_l}{\mu_l}\left(\sum_{m=1}^{M}\mathrm{Tr}(\mathbf{C}_{lm}\mathbf{Q}_m)\right)$$

Extracts variables $\mathbf{Q}_m$'s

[2] Y. Shi, J. Zhang, and K. B. Letaief, "Robust group sparse beamforming for multicast green Cloud-RAN with imperfect CSI," *IEEE Trans. Signal Process.*, vol. 63, no. 17, pp. 4647-4659, Sept. 2015.

Conclusions and Extensions (I)

- **Network power minimization:** A difficult non-convex mixed combinatorial optimization problem

- **Key techniques:**
  - Convexify the combinatorial composite network power consumption function using the mixed ℓ_1/ℓ_2-norm
  - Smoothing the non-smooth group sparsity inducing norm via quadratic variational formulation

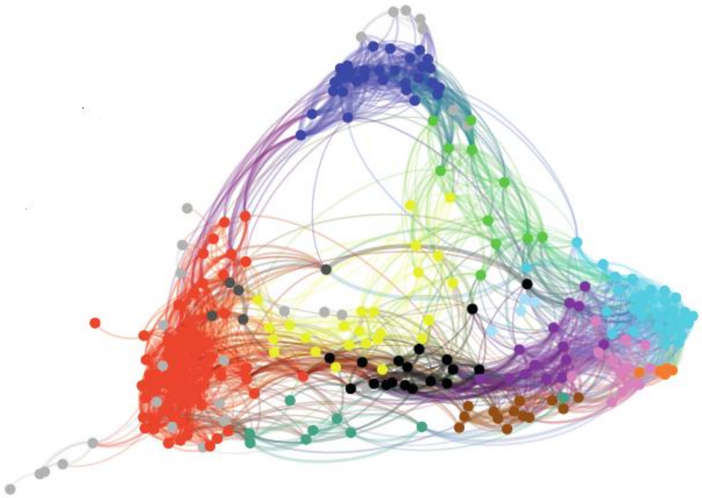
- **Results: Group sparse optimization** offers a principled way to design a green Cloud-RAN

Conclusions and Extensions (II)

- **Extensions:**
  - **User admission [7]:** Smoothed L_p-minimization
  - Limited fronthaul link capacity, CSI uncertainty…
  - Establish the optimality for the group sparse beamforming algorithms
  - More applications in 5G system design, e.g., wireless caching

[7] Y. Shi, J. Cheng, J. Zhang, B. Bai, W. Chen and K. B. Letaief, "Smoothed L_p-minimization for green Cloud-RAN with user admission control," submitted *to IEEE J. Select. Areas Commun.*, under second-round revision.

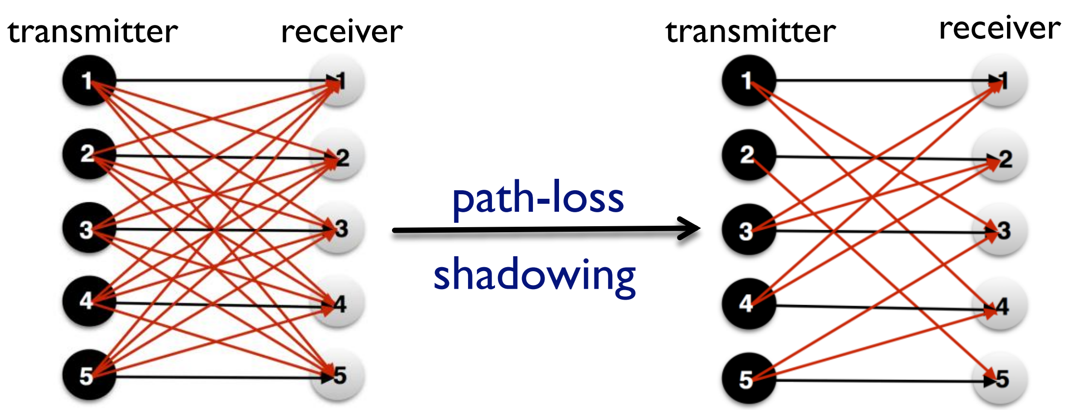
Vignette B: Chance Constrained Optimization for Partially Connected Cloud-RAN



Issue B: Massive Channel State Information

- **Goal:** Interference coordination in dense Cloud-RAN
- **Prior works:** Perfect CSIT [Cadambe and Jafar, TIT 08], delayed CSIT [Maddah-Ali and Tse, TIT 12], alternating CSIT [Tandon, et al., TIT 13],…

- **Curses:** CSIT is rarely abundant (due to training & feedback overhead)
- **Blessings:** Partial connectivity in dense wireless networks [Ruan, et al. TSP 11], [Jafar, TIT 14]

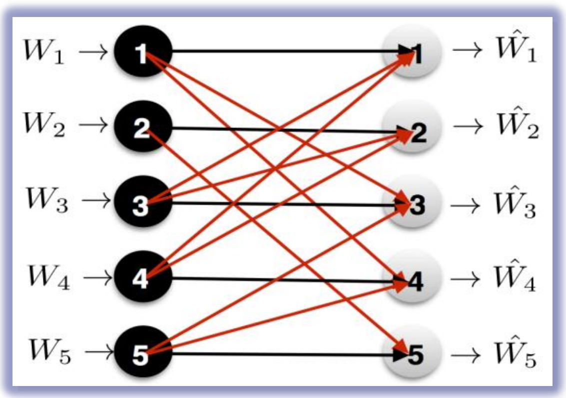


How to exploit the partial connectivity?

Example: TIM via LRMC

- Low-rank matrix completion for topological interference management

transmitters receivers



IA

transmitters

receivers

1		0	0	
	1	0	0	
0		1		0
0			1	0
	0			1

associated incomplete matrix

LRMC

DoF=1/rank

1	.1	0	0	9.5
6.8	1	0	0	64
0	.1	1	-1	0
0	-.1	-1	1	0
.1	0	-.1	.1	1

TIM [Jafar, TIT 14]: Maximize the achievable DoF only based on the network topology information (no CSIT)

Formal Formulation

- **Goal:** Deliver one data stream per user over N time slots
  - $\mathbf{v}_i \in \mathbb{C}^N$: tx. beamformer at the i-th tx.
  - $\mathbf{u}_j \in \mathbb{C}^N$: rx. beamformer at the j-th rx.

Align interference

- **We need:** $X_{ij} = \begin{cases} \mathbf{u}_i^{\mathsf{H}}\mathbf{v}_i = 1, & \forall i, \\ \mathbf{u}_i^{\mathsf{H}}\mathbf{v}_j = 0, & \forall i \neq j, (i,j) \in \Omega, \\ \star, & \text{otherwise.} \end{cases}$ $\xrightarrow{\text{rewrite}}$ $\mathcal{P}_\Omega(\mathbf{X}) = \mathbf{I}_K$ $\Downarrow$ **$1/N$ DoF**

- **Approach:** Low-rank matrix completion (LRMC) [3]

$$\begin{array}{ll} \text{minimize} & \text{rank}(\mathbf{X}) \\ \text{subject to} & \mathcal{P}_\Omega(\mathbf{X}) = \mathbf{I}_K \end{array}$$

Key conclusion: DoF $= 1/\text{rank}(\mathbf{X})$

Any network topology: Ω

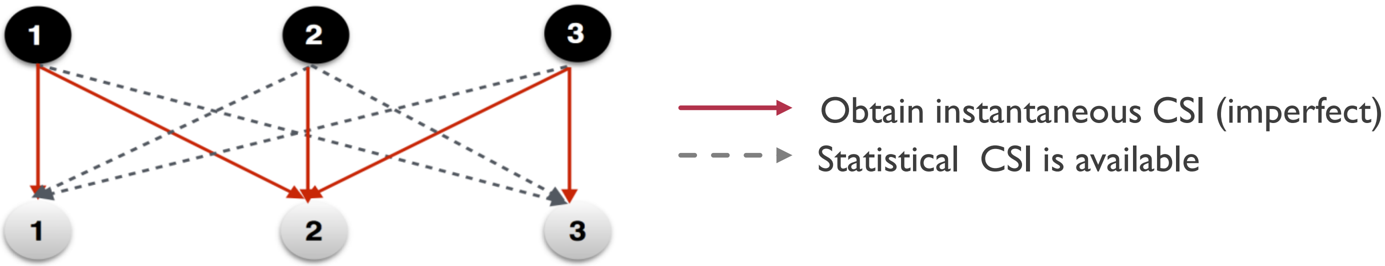
[3] Y. Shi, J. Zhang, and K. B. Letaief, "Low-rank matrix completion via Riemannian pursuit for topological interference management," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT)*, Hong Kong, Jun. 2015.

CSI Uncertainty

- **Uncertainty** in the available CSI
  - Downlink training based channel estimation
  - Uplink limited feedback
  - Hardware deficiencies

How to deal with the CSI uncertainty?

- **Example:** Compressive CSI acquisition [8]



[8] Y. Shi, J. Zhang, and K. B. Letaief, "CSI overhead reduction with stochastic beamforming for cloud radio access networks," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Sydney, Australia, Jun. 2014.

Stochastic vs. Robust

- **Stochastic optimization:** Probabilistic QoS constraints [Lau, et al., TSP 13]

$$\Pr\left\{\frac{|\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k|^2}{\sum_{i\neq k}|\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_i|^2+\sigma_k^2}\geq\gamma_k,\forall k\right\}\geq 1-\epsilon$$

Modeling flexibility: Only distribution information of uncertainty is required

- **Robust optimization:** Worst-case QoS constraints [Ottersten, et al., TSP 12]

$$\min_{\mathbf{e}_k^{\mathsf{H}}\boldsymbol{\Sigma}_k\mathbf{e}_k\leq 1}\frac{|(\hat{\mathbf{h}}_k+\mathbf{e}_k)^{\mathsf{H}}\mathbf{v}_k|^2}{\sum_{i\neq k}|(\hat{\mathbf{h}}_k+\mathbf{e}_k)^{\mathsf{H}}\mathbf{v}_i|^2+\sigma_k^2}\geq\gamma_k,\forall k$$

Uncertainty set modeling is challenging; over conservative

Stochastic Coordinated Beamforming

- **Chance constrained programming:**

$$\begin{aligned} &\underset{\mathbf{v}\in\mathcal{V}}{\text{minimize}} \ \sum_{l=1}^{L}\sum_{k=1}^{K}\|\mathbf{v}_{lk}\|^2 \\ &\text{subject to} \ \Pr\left\{\mathsf{SINR}_k(\mathbf{v},\mathbf{h}_k)\geq\gamma_k,\forall k\right\}\geq 1-\epsilon \end{aligned}$$

- **Challenge:** Non-convex chance constraint

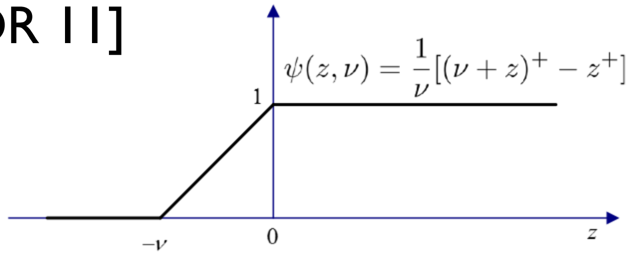
$$\begin{aligned} f(\mathbf{v}) &= 1-\Pr\left\{\mathsf{SINR}_k(\mathbf{v},\mathbf{h}_k)\geq\gamma_k,\forall k\right\} = \Pr\left\{\left(\max_{1\leq k\leq K} d_k(\mathbf{v},\mathbf{h}_k)\right)>0\right\} \\ &= \mathbb{E}\left[1_{(0,+\infty)}\left(\max_{1\leq k\leq K} d_k(\mathbf{v},\mathbf{h}_k)\right)\right] \end{aligned}$$

- **Related works:** Find feasible but sub-optimal solutions
  - Bernstein approximation method (convex relaxation) ([Win, et al., TSP 10], [Lau, et al., TSP 13]): $\exp(z)\geq 1_{(0,+\infty)}(z)$

Sequential Convex Programming

- **Novel approach:** DC (difference-of-convex) function to approximate the indicator function [Hong, et al., OR 11]

$$\psi(z,\nu) = \frac{1}{\nu}[(\nu+z)^+ - z^+], \nu > 0$$



- **DC approximation**:

convex functions

$$\hat{f}(\mathbf{v},\nu) = \mathbb{E}\left[\psi\left(\max_{1\le k\le K} d_k(\mathbf{v},\mathbf{h}_k),\nu\right)\right] = \frac{1}{\nu}[u(\mathbf{v},\nu) - u(\mathbf{v},0)], \nu > 0$$

  - **Sequential convex approximations:** Linearize $\mu(\mathbf{v},0)$

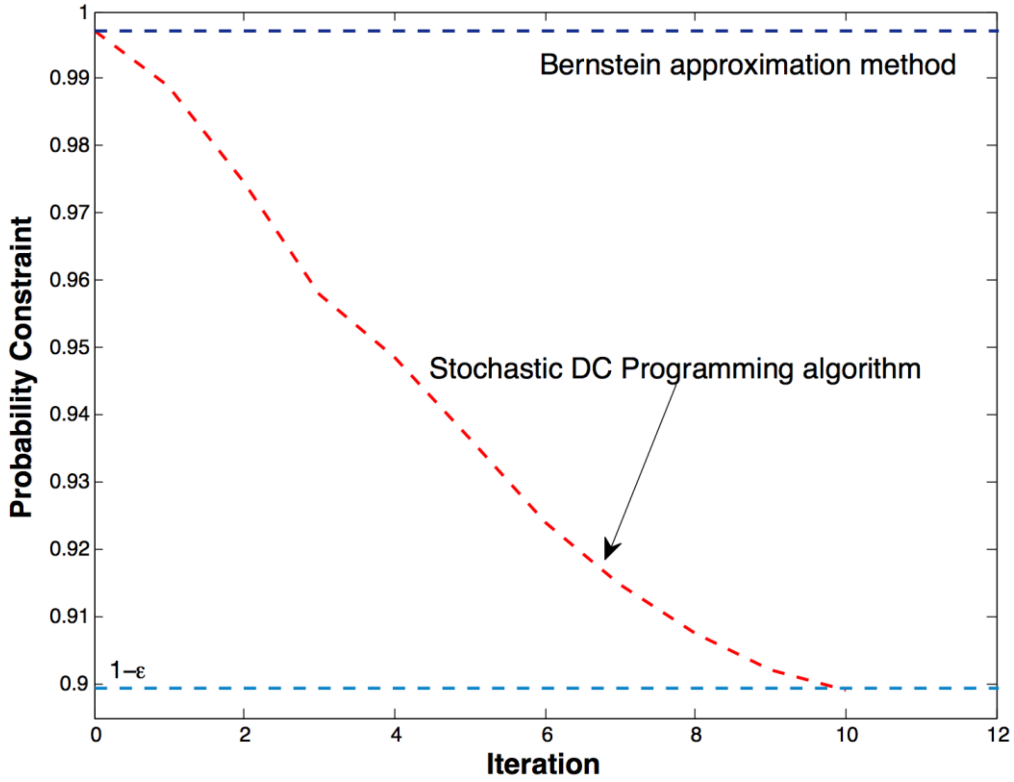
$$l(\mathbf{v};\mathbf{v}^{[j]}) = u(\mathbf{v},\nu) - u(\mathbf{v}^{[j]},0) - 2\langle\nabla_{\mathbf{v}^*}u(\mathbf{v}^{[j]},0), \mathbf{v} - \mathbf{v}^{[j]}\rangle$$

  - **Stochastic DC programming algorithm:** Converge to a KKT point

$$\inf_{\nu>0}\hat{f}(\mathbf{v},\nu) = f(\mathbf{v})$$

Simulation Results (I)

- Conservativeness of approximating probability constraints in the SCB problem (5 RRHs and 3 MUs)

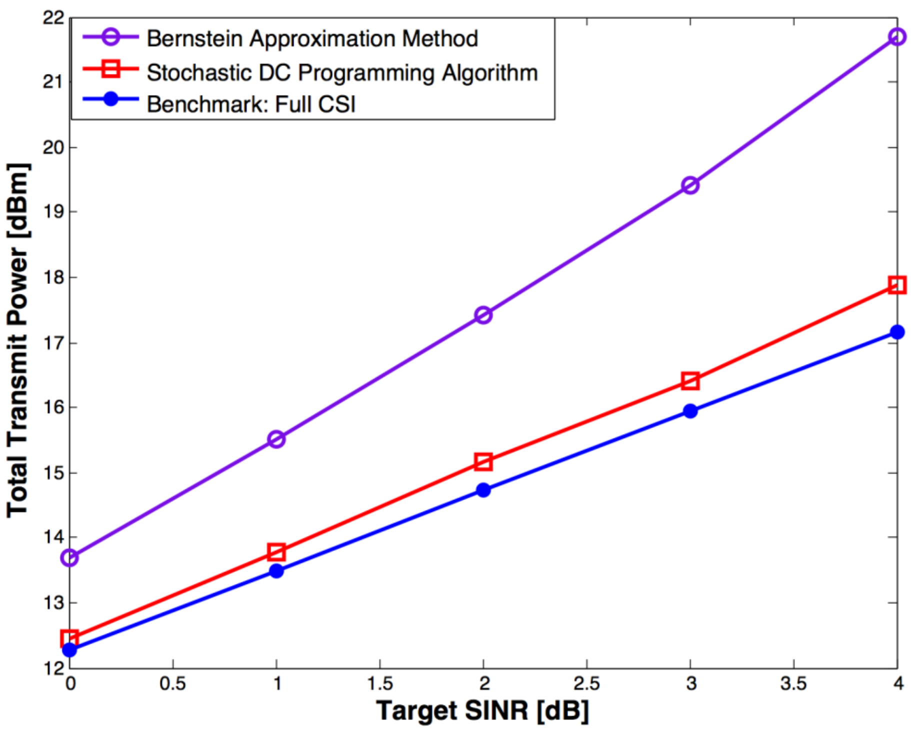


Conservative approximations to the probability constraint

Become tight for the probability constraint

Simulation Results (II)

- Total transmit power versus different target SINR requirements
  - 5 RRHs and 3 MUs, instantaneous CSI 9 out of 15 channel links are obtained



$\mathcal{O}(n^{4.5}\log(1/\epsilon))$

$\mathcal{O}(n^{3.5}MJ\log(1/\epsilon))$

$\mathcal{O}(n^{3.5}\log(1/\epsilon))$

Insights: CSI acquisition overhead can be scalable to large-scale networks due to the partial connectivity of wireless networks.

Conclusions and Extensions (I)

- **Partial connectivity** provides great opportunities for massive CSI overhead reduction
- **New optimization method** is needed to exploit channel structures

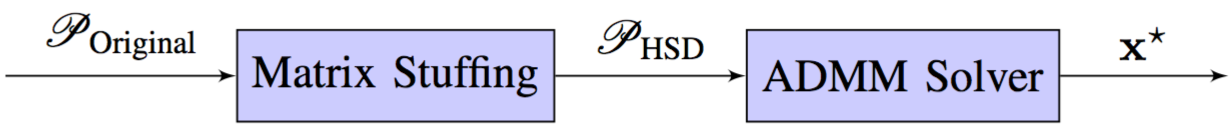
- **Key techniques:**
  - Low-rank matrix completion for topological interference management
  - Sequential convex programming for stochastic coordinated beamforming

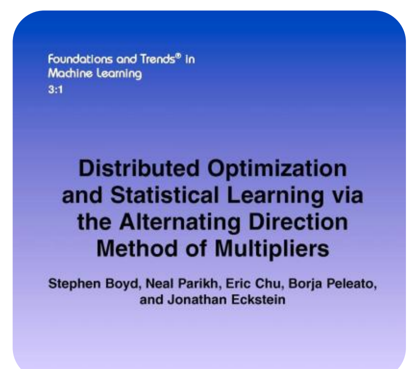
- **Results:**
  - LRMC investigates the TIM problem for **any network topology**
  - SCB provides **modeling flexibility** in the channel knowledge uncertainty

Conclusions and Extensions (II)

- **Extensions:**
  - TIM for partially connected MIMO interference channels
  - Channel estimation by exploiting the channel partial connectivity
  - Improve the computational efficiency for the low-rank matrix completion and stochastic coordinated beamforming problems

Vignette C: ***Large-Scale Convex Optimization*** *for* ***Dense Cloud-RAN***




Foundations and Trends® in
Machine Learning
3:1

Distributed Optimization
and Statistical Learning via
the Alternating Direction
Method of Multipliers

Stephen Boyd, Neal Parikh, Eric Chu, Borja Peleato,
and Jonathan Eckstein

Issue C: Large-Scale Convex Optimization

- **Large-scale convex optimization:** A powerful tool for system design in dense wireless networks

Group sparse beamforming, stochastic beamforming, etc.

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 63, NO. 18, SEPTEMBER 15, 2015

Large-Scale Convex Optimization for Dense Wireless Cooperative Networks

Yuanming Shi, *Student Member, IEEE*, Jun Zhang, *Member, IEEE*, Brendan O'Donoghue, and Khaled B. Letaief, *Fellow, IEEE*

- **Prior works:** Mainly focus on small-size networks or well-structured problems
  - Limitations: scalability [Luo, et al., SPMag 10], parallelization [Yu and Lan, TWC 10], infeasibility detection [Liao, et al., TSP 14], …
- **Unique challenges in dense Cloud-RAN:**
  - Design problems: 1) A high dimension; 2) a large number of constraints; 3) complicated structures

Matrix Stuffing and Operator Splitting

- **Goal:** Design a unified framework for general large-scale convex optimization problem $\mathscr{P}_{\text{Original}}$?
- **Disciplined convex programming framework** [Grant & Boyd '08]

$$\xrightarrow{\mathscr{P}_{\text{Original}}} \boxed{\text{CVX}} \xrightarrow{\mathscr{P}_{\text{cone}}} \boxed{\text{Interior-Point Solver}} \xrightarrow{\mathbf{x}^{\star}}$$

CVX RESEARCH

Time consuming: modeling phase & solving phase

- **Proposal:** Two-stage approach for large-scale convex optimization

$$\xrightarrow{\mathscr{P}_{\text{Original}}} \boxed{\text{Matrix Stuffing}} \xrightarrow{\mathscr{P}_{\text{HSD}}} \boxed{\text{ADMM Solver}} \xrightarrow{\mathbf{x}^{\star}}$$

  - Matrix stuffing: Fast homogeneous self-dual embedding (HSD) transformation
  - Operator splitting (ADMM): Large-scale homogeneous self-dual embedding

Stage One: Fast Transformation

- **Example:** Coordinated beamforming problem family (with transmit power constraints and QoS constraints)

$$\begin{aligned}
\mathscr{P}_{\text{Original}} : \text{minimize} \quad & \|\mathbf{v}\|_2^2 \\
\text{subject to} \quad & \|\mathbf{D}_l \mathbf{v}\|_2 \le \sqrt{P_l}, l = 1, \ldots, L, & (1) \\
& \|\mathbf{C}_k \mathbf{v} + \mathbf{g}_k\|_2 \le \beta_k \mathbf{r}_k^T \mathbf{v}, k = 1, \ldots, K. & (2)
\end{aligned}$$

- **Smith form reformulation** [Smith '96]
  - Key idea: Introduce a new variable for each subexpression in $\mathscr{P}_{\text{Original}}$

Smith form for (1) $\mathcal{G}_1(l) : \begin{cases} (y_0^l, \mathbf{y}_1^l) \in \mathcal{Q}^{KN_l+1} \\ y_0^l = \sqrt{P_l} \in \mathbb{R} \\ \mathbf{y}_1^l = \mathbf{D}_l \mathbf{v} \in \mathbb{R}^{KN_l} \end{cases}$

Second-order cone

Linear constraint

The Smith form is ready for standard cone programming transformation

Stage One: Fast Transformation

- **HSD embedding** of the **primal-dual pair** of transformed standard cone program (based on KKT conditions)

$$\boxed{\begin{aligned} &\underset{\boldsymbol{\nu},\boldsymbol{\mu}}{\text{minimize}} \ \mathbf{c}^T\boldsymbol{\nu} \\ &\text{subject to} \ \ \mathbf{A}\boldsymbol{\nu}+\boldsymbol{\mu}=\mathbf{b} \\ &\qquad\qquad (\boldsymbol{\nu},\boldsymbol{\mu})\in\mathbb{R}^n\times\mathcal{K}. \end{aligned}} + \boxed{\begin{aligned} &\underset{\boldsymbol{\eta},\boldsymbol{\lambda}}{\text{maximize}} \ -\mathbf{b}^T\boldsymbol{\eta} \\ &\text{subject to} \ -\mathbf{A}^T\boldsymbol{\eta}+\boldsymbol{\lambda}=\mathbf{c} \\ &\qquad\qquad (\boldsymbol{\lambda},\boldsymbol{\eta})\in\{0\}^n\times\mathcal{K}^* \end{aligned}} \Longrightarrow \boxed{\begin{aligned} &\mathscr{F}_{\text{HSD}}: \text{find} \ (\mathbf{x},\mathbf{y}) \\ &\text{subject to} \ \mathbf{y}=\mathbf{Q}\mathbf{x} \\ &\qquad\qquad \mathbf{x}\in\mathcal{C},\mathbf{y}\in\mathcal{C}^* \end{aligned}}$$

$$\underbrace{\begin{bmatrix}\boldsymbol{\lambda}\\ \boldsymbol{\mu}\\ \kappa\end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix}\mathbf{0} & \mathbf{A}^T & \mathbf{c}\\ -\mathbf{A} & \mathbf{0} & \mathbf{b}\\ -\mathbf{c}^T & -\mathbf{b}^T & \mathbf{0}\end{bmatrix}}_{\mathbf{Q}} \underbrace{\begin{bmatrix}\boldsymbol{\nu}\\ \boldsymbol{\eta}\\ \tau\end{bmatrix}}_{\mathbf{x}}$$

Certificate of infeasibility: $\tau=0,\kappa>0$

- **Matrix stuffing for fast transformation:**
  - Generate and keep the structure $\mathbf{Q}$
  - Copy problem instance parameters to the pre-stored structure $\mathbf{Q}$

Stage Two: Parallel and Scalable Computing

- **HSD embedding in consensus form:**

$$\begin{array}{lll} \mathscr{F}_{\mathrm{HSD}}: \text{find} & (\mathbf{x},\mathbf{y}) \\ \quad \text{subject to} & \mathbf{y} = \mathbf{Q}\mathbf{x} \\ & \mathbf{x}\in\mathcal{C}, \mathbf{y}\in\mathcal{C}^* \end{array} \rightarrow \begin{array}{ll} \mathscr{P}_{\mathrm{ADMM}}: \underset{\mathbf{x},\tilde{\mathbf{x}},\mathbf{y},\tilde{\mathbf{y}}}{\text{minimize}} & I_{\mathcal{C}\times\mathcal{C}^*}(\mathbf{x},\mathbf{y}) + I_{\mathbf{Q}\tilde{\mathbf{x}}=\tilde{\mathbf{y}}}(\tilde{\mathbf{x}},\tilde{\mathbf{y}}) \\ \quad \text{subject to} & (\mathbf{x},\mathbf{y}) = (\tilde{\mathbf{x}},\tilde{\mathbf{y}}) \end{array}$$

- **Final algorithm:** Apply the operating splitting method (ADMM) [Donoghue, Chu, Parikh, and Boyd '13]

$$\begin{aligned} \tilde{\mathbf{x}}^{[i+1]} &= (\mathbf{I}+\mathbf{Q})^{-1}(\mathbf{x}^{[i]}+\mathbf{y}^{[i]}) && \text{subspace projection} \\ \mathbf{x}^{[i+1]} &= \Pi_{\mathcal{C}}(\tilde{\mathbf{x}}^{[i+1]}-\mathbf{y}^{[i]}) && \text{parallel cone projection} \\ \mathbf{y}^{[i+1]} &= \mathbf{y}^{[i]}-\tilde{\mathbf{x}}^{[i+1]}+\mathbf{x}^{[i+1]} && \text{computationally trivial} \end{aligned}$$

- **Proximal algorithms for parallel cone projection** [Parikn & Boyd, FTO 14]
  - E.g., Projection onto the second-order cone $\mathcal{C}_i = \{(y,\mathbf{x})\in\mathbb{R}\times\mathbb{R}^{p-1} | \|\mathbf{x}\|\le y\}$

$$\Pi_{\mathcal{C}_i}(\boldsymbol{\omega},\tau) = \begin{cases} 0, \|\boldsymbol{\omega}\|_2 \le -\tau \\ (\boldsymbol{\omega},\tau), \|\boldsymbol{\omega}\|_2 \le \tau \\ (1/2)(1+\tau/\|\boldsymbol{\omega}\|_2)(\boldsymbol{\omega},\|\boldsymbol{\omega}\|_2), \|\boldsymbol{\omega}\|_2 \ge |\tau|. \end{cases}$$

Numerical Results (I)

- **Example:** Power minimization coordinated beamforming problem [6]

Network Size (*L*=*K*)		20	50	100	150
CVX+SDPT3	Modeling Time [sec]	**0.7563**	4.4301	N/A	N/A
	Solving Time [sec]	4.2835	**326.2513**	N/A	N/A
	Objective [W]	12.2488	6.5216	N/A	N/A
Matrix Stuffing+ADMM	Modeling Time [sec]	**0.0128**	0.2401	2.4154	9.4167
	Solving Time [sec]	0.1009	**2.4821**	23.8088	81.0023
	Objective [W]	12.2523	6.5193	3.1296	2.0689

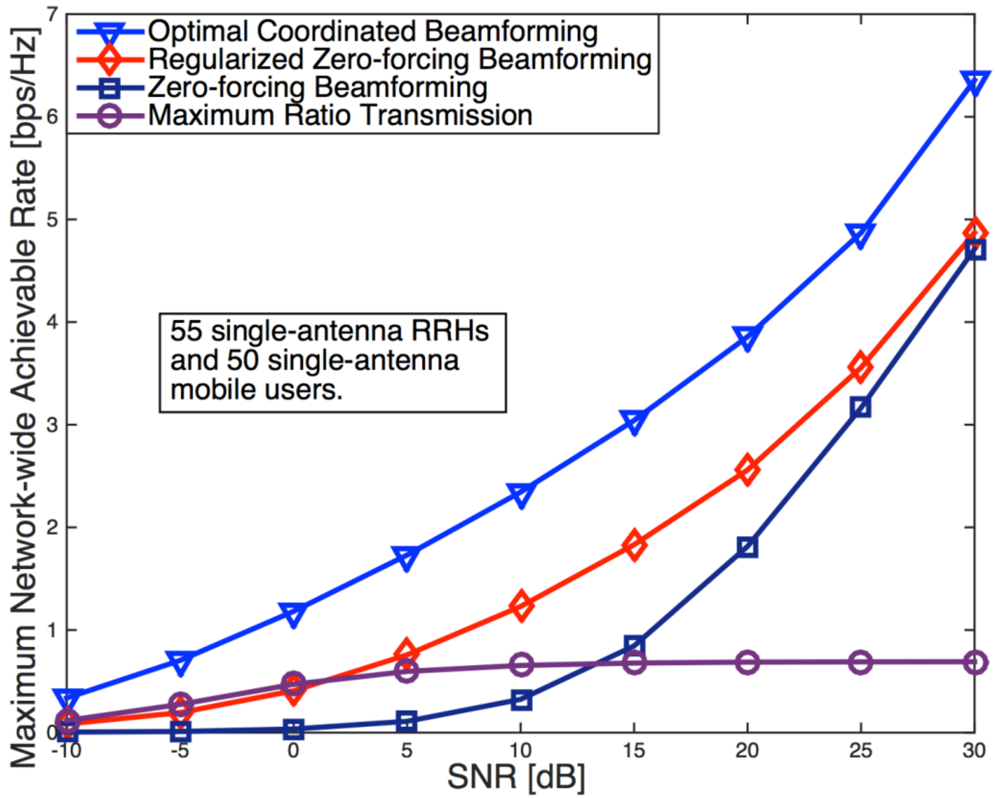
Matrix stuffing can speedup **60x** over CVX

ADMM can speedup **130x** over the interior-point method

[6] Y. Shi, J. Zhang, B. O'Donoghue, and K. B. Letaief, "Large-scale convex optimization for dense wireless cooperative networks," *IEEE Trans. Signal Process.*, vol. 63, no. 18, pp. 4729-4743, Sept. 2015.

Numerical Results (II)

- Coordinated beamforming for max-min fairness rate optimization [6]



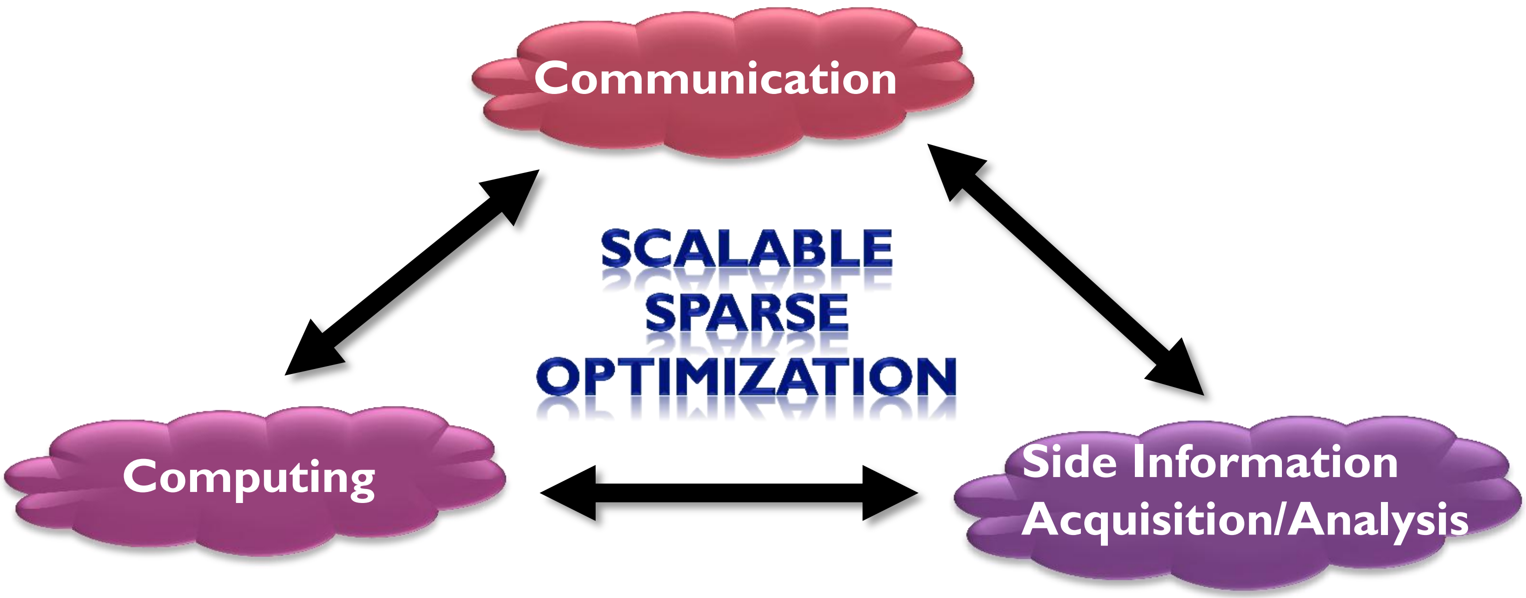
Large-scale optimal coordinated beamforming is needed for dense Cloud-RAN

Conclusions and Extensions

- **Large-scale convex optimization** is essential to enable scalability and flexibility in dense Cloud-RAN
- **Key techniques:**
  - Matrix stuffing: Fast transformation
  - Operator splitting method (ADMM): Large-scale HSD embedding
- **Results: Two-stage large-scale optimization framework** provides a unified way to solve general large-scale convex programs **in parallel**
- **Extensions:**
  - Parallel and distributed implementations (Hadoop, Spark)
  - Randomized algorithms for the semidefinite cone projection (SDP problems)

Summary (I)

- **The following interaction becomes more and more important:**



Summary (II)

- **Cloud radio access network** is an enabling architecture that allows
  - Joint signal processing across the network
  - Advanced network-wide optimization in the cloud

- **Summary of results:**
  - Group sparse optimization enables flexible network adaptation
  - Partial connectivity provides opportunities for CSI overhead reduction
  - LRMC and stochastic optimization are powerful to exploit channel structures
  - Large-scale convex optimization plays a key role in network optimization

Future network design: Dense, cooperative, scalable, unified

Further Information: Journal Articles

- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "Low-rank matrix completion for topological interference management by Riemannian pursuit," submitted to *IEEE Trans. Wireless Commun*., Jul. 2015.
- **<u>Y. Shi</u>**, J. Cheng, J. Zhang, B. Bai, W. Chen and K. B. Letaief, "Smoothed L_p-minimization for green Cloud-RAN with user admission control," submitted to *IEEE J. Select. Areas Commun*., under second-round revision.
- **<u>Y. Shi</u>**, J. Zhang, B. O'Donoghue, and K. B. Letaief, "Large-scale convex optimization for dense wireless cooperative networks," *IEEE Trans. Signal Process*., vol. 63, no. 18, pp. 4729-4743, Sept. 2015.
- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "Robust group sparse beamforming for multicast green Cloud- RAN with imperfect CSI," *IEEE Trans. Signal Process*., vol. 63, no. 17, pp. 4647-4659, Sept. 2015.
- **<u>Y. Shi</u>**, J. Zhang, K. B. Letaief, B. Bai and W. Chen,"Large-scale convex optimization for ultra-dense Cloud-RAN," *IEEE Wireless Commun. Mag*., pp. 84-91, Jun. 2015.
- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "Optimal stochastic coordinated beamforming for wireless cooperative networks with CSI uncertainty," *IEEE Trans. Signal Process*., vol. 63,, no. 4, pp. 960-973, Feb. 2015.
- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "Group sparse beamforming for green Cloud-RAN," *IEEE Trans. Wireless Commun*., vol. 13, no. 5, pp. 2809-2823, May 2014.

Further Information: Conference Papers

- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "Low-rank matrix completion via Riemannian pursuit for topological interference management," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT),* Hong Kong, Jun. 2015.
- J. Cheng, **<u>Y. Shi</u>**, B. Bai, W. Chen, J. Zhang, and K. B. Letaief, "Group sparse beamforming for multicast green Cloud-RAN via parallel semidefinite programming," in *Proc. IEEE Int. Conf. Commun. (ICC),* London, UK, Jun. 2015.
- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "Scalable coordinated beamforming for dense wireless cooperative networks," in *Proc. IEEE Globecom*, Austin, TX, Dec. 2014.
- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "CSI overhead reduction with stochastic beamforming for cloud radio access networks," in *Proc. IEEE Int. Conf. Commun. (ICC),* Sydney, Australia, Jun. 2014.
- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "*Group sparse beamforming for green cloud radio access net- works*," in *Proc. IEEE Globecom*, Atlanta, GA, Dec. 2013.
- **<u>Y. Shi</u>**, J. Zhang, and K. B. Letaief, "Coordinated relay beamforming for amplify-and-forward two-hop interference networks," in *Proc. IEEE Globecom*, Anaheim, CA, Dec. 2012.

Thanks